$\boxed{\text{Local optimizer}}$

Step 1 $(x^*, y^*) \rightarrow \nabla F = 0$
$$\left.\begin{array}{l} \nabla F = 0 \\ \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \end{array}\right\} (x^*, y^*) \Downarrow \text{stationary point}$$

<u>Step 2</u>: f: Convex/Concave (x^*, y^*)

$d^2 f > 0$ (convex) $\rightarrow |H_1| > 0$; $|H_2| > 0$

< 0 (concave) $\rightarrow |H_1| < 0$; $|H_2| > 0$

Pattern doesn't match $\rightarrow$ ~~no~~ inconclusive

<u>Global</u>: local $\rightarrow$ global ; globally concave $\forall (x$ negative

globally convex $\forall (x$ positive

II

Consider the following:

$$y = -2x_1^2 + 4x_1x_2 - 5x_2^2 + 2x_2x_3 - 3x_3^2 + 2x_1x_3$$

Find optimizing point(s).

<u>Step1</u> $\underline{(x_1^*, x_2^*, x_3^*)}$: stationary point.

$$\left.\begin{aligned} f_1 &= -4x_1 + 4x_2 + 2x_3 = 0 \quad (1) \\ f_2 &= 4x_1 - 10x_2 + 2x_3 = 0 \quad (2) \\ f_3 &= 2x_2 - 6x_3 + 2x_1 = 0 \quad (3) \end{aligned}\right\} (x_1^*, x_2^*, x_3^*)$$

$$\Rightarrow \begin{pmatrix} -4 & 4 & 2 \\ 4 & -10 & 2 \\ 2 & 2 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 = 0$, $x_2 = 0$, $x_3 = 0$ } "Stationay" point

(Trivial Sol^n)

max / min

$$H = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} = \begin{pmatrix} -4 & 4 & 2 \\ 4 & -10 & 2 \\ 2 & 2 & -6 \end{pmatrix}$$

$$|H_1| = |-4| = -4 < 0$$

$(\underline{0}, \underline{0}, \underline{0}) \rightarrow$ local max

$\rightarrow$ global m

$$|H_2| = \begin{vmatrix} -4 & 4 \\ 4 & -10 \end{vmatrix} = (-4)(-10) - (4)(4) = 40 - 16 = 24 > 0$$

$$|H_3| = \overset{-240}{(-4)(-10)(-6)} + \overset{16}{(4)(2)}\boxed{(2)} + \overset{16}{(2)(4)(2)}$$

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$$\underset{+40}{-(2)(-10)(2)} \underset{+16}{- (2)(2)(-4)} \underset{+96}{- (-6)(4)(4)}$$

$$-240 + 188 = -62 < 0$$ H is negative definite

globally concave!

$Q = \sqrt{K} + \sqrt{L}$ "p" ; "w" ; "r"

① $\max \pi = p \cdot (\sqrt{K} + \sqrt{L}) - (wL + rK)$

(K^*, L^*) ✓ $\max \pi$

$K^* = (P/2r)^2$

$L^* = (P/2W)^2$

local max ⇓ global max

② Apply F.O.C / S.O.C method.

F.O.C.

$\frac{\partial \pi}{\partial K} = p \cdot \left(\frac{1}{2} \cdot K^{-\frac{1}{2}}\right) - r = 0$

$\frac{p}{2} \cdot K^{-\frac{1}{2}} = r$

$\frac{p}{2r} = K^{\frac{1}{2}}$

$K = \left(\frac{p}{2r}\right)^2$

$\frac{\partial \pi}{\partial L} = p \cdot \left(\frac{1}{2} L^{-\frac{1}{2}}\right) - w = 0$

$(K^*, L^*) = \cancel{\left(\frac{p}{2w}\right.} \left[\left(\frac{p}{2r}\right)^2, \left(\frac{p}{2w}\right)^2\right]$

$$\left\{ K^* = \left(\frac{P}{2r}\right)^2 \; ; \; L^* = \left(\frac{P}{2w}\right)^2 \right\}$$ Capital is cheap $\to K^* > L^*$

Demand for K   Remand for L

$P = 1{,}000 \; ; \; w = 20 \; ; \; r = 10$

$$K^* = \left(\frac{1000}{2 \cdot 10}\right)^2 = (50)^2 = 2{,}500$$ $K^* > L^*$

$$L^* = \left(\frac{1000}{2 \cdot 20}\right)^2 = (25)^2 = 625$$ $r < w$

$Q = \sqrt{K} + \sqrt{L}$ Perfectly Competitive

Problem. / Objective: $\max \pi = \underbrace{P(\sqrt{K}+\sqrt{L})}_{TR} - \underbrace{\overset{\text{Labor}}{WL} - \overset{\text{Capita}}{rK}}_{TC}$

w = Wage
r = rent
p = Price
} given.

Factor input decision. F.O.C. $VMP_L = w$; $VMP_K = r$

D for capital: $\boxed{K^* = \left(\frac{P}{2r}\right)^2}$; $\frac{\partial K^*}{\partial P} = 2P \cdot \left(\frac{1}{2r}\right)^2 > 0$; $\frac{\partial K^*}{\partial r} = -2r^{-3} \cdot \left(\frac{P}{2}\right)^2 <$

D for labor: $\boxed{L^* = \left(\frac{P}{2w}\right)^2}$; $\frac{\partial L^*}{\partial P} = 2P \left(\frac{1}{2w}\right)^2 > 0$; $\frac{\partial K^*}{\partial w} = -2w^{-3}\left(\frac{P}{2}\right)^2$

P↑ ; factor inputs ↑ (both)

No Cross price Effect : r does n't affect L^*.
w " " K^*.

(1)

Optimal Profit function-

$$K^*, L^* \Rightarrow \Pi$$

$$\Pi^* = p\left(\sqrt{K^*} + \sqrt{L^*}\right) - rK^* - wL^*.$$

$$= p\left(\frac{p}{2r} + \frac{p}{2w}\right) - r\left(\frac{p}{2r}\right)^2 - w\left(\frac{p}{2w}\right)^2$$

$$= \frac{p^2}{2r} + \frac{p^2}{2w} - \frac{1}{4}\frac{p^2}{r} - \frac{1}{4}\frac{p^2}{w}$$

$$= \frac{1}{4}\frac{p^2}{2r} + \frac{1}{4}\frac{p^2}{2w} = \Pi^*(p, r, w) = \text{optimized profit function}$$

$\Rightarrow f^2$ that describes "<u>behaviour</u>" (value) of the <u>maximized level</u> of profit

F.O.C.: "f" any arbitrary Production f^n

$$\begin{aligned}
&[K] \quad p \cdot f_K(K^*, L^*) = r \quad — ① \\
&[L] \quad p \cdot f_L(K^*, L^*) = w \quad — ②
\end{aligned} \Bigg\} \quad \begin{aligned} K^* &= K^*(p, w, r) \\ L^* &= L^*(p, w, r). \end{aligned}$$

$$(dK^*, dL^*) = ?$$

$$d① \Rightarrow d[p \cdot f_K(K^*, L^*)] = dr$$

$$f_K \cdot dp + p \cdot df_K = dr$$

$$\hookrightarrow df_K = f_{KK} dK + f_{KL} \cdot dL$$

$$* \quad \left(f_K dp + (f_{KK} dK + f_{KL} dL) \cdot p\right) = dr \quad — ①$$ in the total diff form

②

$$d(p f_L(k^*, L^*)) = dw$$

$$** \quad \overset{f_L}{\cancel{f_K}}\, dp + p\left[f_{LK}\, dk + f_{LL}\, dL\right] = dw \quad — ②$$

in the total diff for

$$\begin{pmatrix} p f_{KK} & p f_{KL} \\ p f_{LK} & p f_{LL} \end{pmatrix} \begin{pmatrix} dk \\ dL \end{pmatrix} = \begin{pmatrix} dr - f_K\, dp \\ dw - f_L \cdot dp \end{pmatrix}$$

$$dk = \frac{\begin{vmatrix} dr - f_K\, dp & p \cdot f_{KL} \\ dw - f_L\, dp & p \cdot f_{LL} \end{vmatrix}}{p \begin{vmatrix} f_{KK} & f_{KL} \\ f_{LK} & f_{LL} \end{vmatrix}}$$

$$dK = \frac{(\underline{dr} - f_K \cdot \underline{dp})\, p \cdot f_{LL} - p f_{KL} (dw - f_L \underline{dp})}{\underbrace{\underset{+}{p} \cdot [\underset{+}{f_{KK} \cdot f_{LL}} - (f_{KL})^2]}_{+}}$$

A $\rightarrow$

f: is concave

$$H \text{ production } f^2 = \begin{bmatrix} f_{KK} & f_{KL} \\ f_{LK} & f_{LL} \end{bmatrix} \quad f_{KK} < 0 \;;\; f_{LL} <$$

$$|H| = f_{KK} \cdot f_{LL} - (f_{KL})^2 > 0$$

Top tems:

$$p\cdot f_{LL} dr - f_K \cdot p \cdot f_{LL} \cdot dp + p \cdot f_{KL} \cdot f_L \cdot dp$$

$$- p \cdot f_{KL} dw$$

$$p \cdot f_{LL} dr + (p \cdot f_{KL} \cdot f_L - f_K \cdot p \cdot f_{LL}) \cdot dp - p \cdot f_{KL} dw$$

$$dk = \frac{p \cdot f_{LL}}{A} \cdot dr + \frac{(p \cancel{f} f_{KL} f_L - f_K \cdot p \cdot f_{LL})}{A} dp$$

$$- \frac{p \cdot f_{KL}}{A} \cdot dw$$

"Baseline"

Single Prize
Perfect Comp. ⟶ Market structure ⟶ Monopoly
⟶ monopolistically Competition.
⟶ few firms ⟶ Cournot problem
with identical goods

Two factor ⟶ Multi (more) factors (K, L, gas, Energy, land, material

One Product ⟶ Multi products ⟶ e.g. Unilivers : household products
wide variety

One plant. ⟶ multi plant ⟶ e.g. plants located in different locations
(tax treatment

historical reason. { taking over
merger

Economy of Scale

$P = 25$

$TC(Q_1) = 2Q_1^2 + 5Q_1 + 10$

$TC(Q_2) = 2Q_2^2 + 3Q_2 + 15$

$\rightarrow Q \begin{cases} Q_1 \\ Q_2 \end{cases} \Big\} \ \underline{Q = Q_1 + Q_2}$

$\pi_{(Q_1, Q_2)} = (25)(Q_1 + Q_2) - (2Q_1^2 + 5Q_1 + 10) - (2Q_2^2 + 3Q_2 + 15)$

Q in 1st plant

Q in 2nd plant

F.O.C.

$[Q_1] \Rightarrow \frac{\partial \pi}{\partial Q_1} = 25 - (4Q_1 + 5) = 0 \rightarrow \underline{Q_1^* = 5}$

$[Q_2] \Rightarrow \frac{\partial \pi}{\partial Q_2} = 25 - (4Q_2 + 3) = 0 \rightarrow \underline{Q_2^* = 5.5}$

S.O.C.

$$H = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$|H_1| = -4 \; ; \; |H_2| = (-4)(-4) = 16 > 0$

globally concave

$(5, 5.5)$ global maximizer!

$$P = 10 - Q$$

$$TC = 2Q_1^2 + 5Q_1 + 10$$

$$TC = 2Q_2^2 + 3Q_2 + 15.$$

$$\Pi = [10 - (Q_1 + Q_2)](Q_1 + Q_2) - (2Q_1^2 + 5Q_1 + 10)$$

$$- (2Q_1^2 + 3Q_2 + 15)$$

$$\frac{\partial \Pi}{\partial Q_1} = [10 - 2(Q_1 + Q_2)] - (4Q_1 + 5) = 0$$

$$\frac{\partial \Pi}{\partial Q_2} = [10 - 2(Q_1 + Q_2)] - (4Q_2 + 3) = 0$$

$$10 - 2Q_1 - 2Q_2 - 4Q_1 - 5 = 0$$

$$6Q_1 + 2Q_2 = 5 \quad — (3)$$

$$2Q_1 + 6Q_2 = 7 \quad — (4)$$

Solve for Q_1^* and Q_2^* $\rightarrow$ (3) & (4)