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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^{5}(a+bx_i)$ $\sum_{i=1}^{5} a + \sum_{i=1}^{5} bx_i = 5a + b\sum_{i=1}^{5} x_i = 5a + b(x_1+x_2+x_3+x_4+x_5)$

b. $\sum_{y=0}^{5} f(x+y) = f(x+0)+f(x+1)+f(x+2)+f(x+3)+f(x+4)+f(x+5)$

c. $\sum_{i=1}^{10} i^2 = 1^2+2^2+3^2+4^2+5^2 \ldots + 10^2 = 1+4+9+16+25+36+49+68+81+100$

$= 385$

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y) = \sum_{x=1}^{2}(2x+2+2x+3) = \sum_{x=1}^{2}(4x+5) = 4(1)+5+4(2)+5$

$= 22$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$0.5b+b+2.25b+2b+1.5b+0.5b+0.25b = 1$ | $b = 1/8$

b. Find the answer for $P(X \leq 2)$

$P = 1-0.5b-0.25b = 1-0.09375 = 0.9063$

c. Find the answer for $P(-2 \leq X \leq 3)$

$P = 1-0.25b = 1-0.03125 = 0.9688$

d. Find the answer for $P(X \geq 1)$

$P = 2b+1.5b+0.5b+0.25b = 0.5313$

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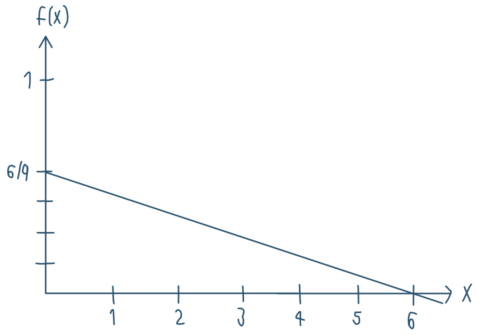
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

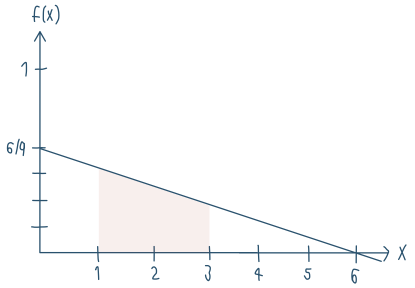
$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

0	1	2	3
6/9	5/9	4/9	3/9

a. Plot graph for $f(x)$



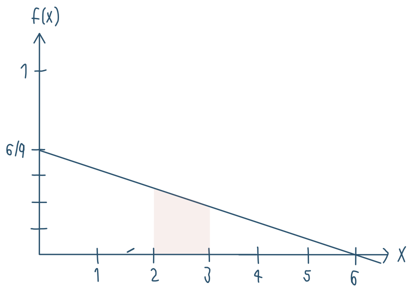
b. Find the answer for $P(1 \leq X \leq 3)$



$$p(1 \leq x < 3) = \int_1^3 -\frac{1}{9}x + \frac{6}{9} = -\frac{1}{18}x^2 + \frac{6x}{9}\Big|_1^3$$

$$= \frac{3}{2} - \frac{11}{18} = \frac{8}{9} = 0.889$$

c. Find the answer for $P(X \geq 2)$



$$P(X \geq 2) = \int_2^3 -\frac{1}{9}x + \frac{6}{9} = -\frac{1}{18}x^2 + \frac{6x}{9}\Big|_2^3$$

$$= \frac{3}{2} - \frac{10}{9} = \frac{7}{18}$$

d. Find the expected value of X

$$\int_0^3 xf(x)dx = \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx = \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x\right)dx$$

$$= \frac{-x^3}{27} + \frac{6x^2}{18}\Big|_0^3$$

$$= -1 + 3$$

$$= 2$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

$1/6 \cdot 1/2$

Y \ X	1	2	3	4	5	6	
0	1/12	1/12	1/12	1/12	1/12	1/12	$\frac{1}{2}$
1	1/12	1/12	1/12	1/12	1/12	1/12	$\frac{1}{2}$
	1/6	1/6	1/6	1/6	1/6	1/6	1

   b. Find the marginal probability distribution function (PDF) of X

$P(X=1) = 1/6$ $\quad P(X=3) = 1/6$ $\quad P(X=5) = 1/6$

$P(X=2) = 1/6$ $\quad P(X=4) = 1/6$ $\quad P(X=6) = 1/6$

   c. Find the marginal probability distribution function (PDF) of Y

$P(Y=0) = 1/2$

$P(Y=1) = 1/2$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X \mid Y=1) = \frac{P(X,Y=1)}{P(Y=1)} = \frac{P(X,Y=1)}{0.5} = 2P(X,Y=1)$$

   e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum_1 x_i P(X=x_i|Y=1) = 1(1/12)+2(1/12)+3(1/12)+4(1/12)+5(1/12)+6(1/12)$$

$$= 21(1/12) = 21/12 = 7/4$$

   f. Find the variance of X given Y is equal to 1

$$var(X|Y=1) = E[X^2|Y=1] - [E(X|Y=1)]^2$$

$$= \sum_i x_i P(X_i|Y=1) - 3.5^2$$

$$= 1^2 \cdot \cancel{2}(\tfrac{1}{\cancel{12}}) + 2^2 \cdot \cancel{2}(\tfrac{1}{\cancel{12}}) + \dots + 6^2(\cancel{2})(\tfrac{1}{\cancel{12}}) - 3.5^2$$

$$= \frac{1}{6}(91) - 3.5^2 = 2.9667$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left[\frac{1}{3}(X_1 + X_2 + X_3)\right]$$

$$E(\bar{X}) = \frac{1}{3}E(X_1 + X_2 + X_3)$$

$$E(\bar{X}) = \frac{1}{\cancel{3}}(\cancel{3}\mu)$$

$$E(\bar{X}) = \mu$$

$$var(\bar{X}) = var\left[\frac{1}{3}(X_1 + X_2 + X_3)\right]$$

$$var(\bar{X}) = \frac{1}{9}var(X_1 + X_2 + X_3)$$

$$var(\bar{X}) = \frac{1}{9}\left(var(X_1) + var(X_2) + var(X_3) + 2cov(X_1X_2) + 2cov(X_2, X_3) + 2cov(X_1, X_3)\right)$$

$$var(\bar{X}) = \frac{1}{9}\left[3\sigma^2 + \frac{3}{4}\sigma^2\right]$$

$$var(\bar{X}) = \frac{\sigma^2}{2}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left[\frac{1}{4}(X_1 + X_2 + X_3 + X_4)\right]$$

$$E(\bar{X}) = \frac{1}{4}\left[\underbrace{E(X_1)}_{\mu} + \underbrace{E(X_2)}_{\mu} + \underbrace{E(X_3)}_{\mu} + \underbrace{E(X_4)}_{\mu}\right]$$

$$E(\bar{X}) = \frac{1}{4}(4\mu)$$

$$E(\bar{X}) = \mu$$

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$$var(\bar{X}) = var\left[\frac{1}{4}(X_1+X_2+X_3+X_4)\right]$$

$$var(\bar{X}) = \frac{1}{16}\left[\overset{\sigma}{var}(X_1)+\overset{\sigma}{var}(X_2)+\overset{\sigma}{var}(X_3)+\overset{\sigma}{var}(X_4)\right]$$

$$var(\bar{X}) = \frac{1}{16}(4\sigma^2)$$

$$var(\bar{X}) = \sigma^2/4$$

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$E(\tilde{X}) = E\left[\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right]$$

$$E(\tilde{X}) = \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4)$$

$$E(\tilde{X}) = \frac{1}{8}\mu + \frac{1}{4}\mu + \frac{1}{8}\mu + \frac{1}{2}\mu$$

$$E(\tilde{X}) = \mu$$

As $E(\tilde{X}) = \mu$, $\tilde{X}$ is an unbiased estimator of μ.

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$$var(\tilde{X}) = var\left[\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right]$$

$$var(\tilde{X}) = \frac{1}{64}var(X_1) + \frac{1}{16}var(X_2) + \frac{1}{64}var(X_3) + \frac{1}{4}var(X_4)$$

$$var(\tilde{X}) = \frac{1}{64}\sigma^2 + \frac{1}{16}\sigma^2 + \frac{1}{64}\sigma^2 + \frac{1}{4}\sigma^2$$

$$var(\tilde{X}) = \frac{11}{32}\sigma^2$$

As $var(\bar{X}) < var(\tilde{X})$, $\bar{X}$ is the better estimator for μ.