

Exercises

- Consider a lottery with three possible outcomes: \$100 will be received with probability .1, \$50 with probability .2, and \$10 with probability .7.
 - What is the expected value of the lottery?
 - What is the variance of the outcomes of the lottery?
 - What would a risk-neutral person pay to play the lottery?
- Suppose you have invested in a new computer company whose profitability depends on (1) whether the U.S. Congress passes a tariff that raises the cost of Japanese computers, and (2) whether the U.S. economy grows slowly or quickly. What are the four mutually exclusive states of the world that you should be concerned about?
- Richard is deciding whether to buy a state lottery ticket. Each ticket costs \$1, and the probability of winning payoffs is given as follows:

Probability	Return
.5	\$0.00
.25	\$1.00
.2	\$2.00
.05	\$7.50

- What is the expected value of Richard's payoff if he buys a lottery ticket? What is the variance?
 - Richard's nickname is "No-risk Rick." He is an extremely risk-averse individual. Would he buy the ticket?
 - Suppose Richard was offered insurance against losing any money. If he buys 1,000 lottery tickets, how much would he be willing to pay to insure his gamble?
 - In the long run, given the price of the lottery ticket and the probability/return table, what do you think the state would do about the lottery?
- Suppose an investor is concerned about a business choice in which there are three

prospects, whose probability and returns are given below:

Probability	Return
0.2	\$100
0.4	50
0.4	-25

What is the expected value of the uncertain investment? What is the variance?

- You are an insurance agent who has to write a policy for a new client named Sam. His company, Society for Creative Alternatives to Mayonnaise (SCAM), is working on a low-fat, low-cholesterol mayonnaise substitute for the sandwich condiment industry. The sandwich industry will pay top dollar to whoever invents such a mayonnaise substitute first. Sam's SCAM seems like a very risky proposition to you. You have calculated his possible returns table as follows:

Probability	Return
.999	-\$1,000,000 (he fails)
.001	\$1,000,000,000 (he succeeds and sells the formula)

- What is the expected return of his project? What is the variance?
- What is the most Sam is willing to pay for insurance? Assume Sam is risk neutral.
- Suppose you found out that the Japanese are on the verge of introducing their own mayonnaise substitute next month. Sam does not know this and has just turned down your final offer of \$1000 for the insurance. If Sam tells you his SCAM is only six months away from perfecting his mayonnaise substitute and knowing what you know about the Japanese, would you raise

- or lower your policy premium on any subsequent proposal to Sam? Based on his information, would Sam accept?
- Suppose that Natasha's utility function is given by $u(I) = y^2$, where I represents annual income in thousands of dollars.
 - Is Natasha risk loving, risk neutral, or risk averse? Explain.
 - Suppose that Natasha is currently earning an income of \$10,000 ($I = 10$) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .5 probability of earning \$16,000, and a .5 probability of earning \$5000. Should she take the new job?
 - In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (*Hint:* What is the risk premium?)
 - Draw a utility function over income $u(I)$ that has the property that a man is a risk lover when his income is low but a risk averter when his income is high. Can you explain why such a utility function might reasonably describe a person's tastes?
 - A city is considering how much to spend monitoring parking meters. The following information is available to the city manager:
 - Hiring each meter-monitor costs \$10,000 per year.
 - With one monitoring person hired, the probability of a driver getting a ticket each time he or she parks illegally is equal to .25.
 - With two monitors hired, the probability of getting a ticket is .5, with three monitors the probability is .75, and with four the probability is equal to 1.
 - The current fine for overtime parking with two metering persons hired is \$20.
 - Assume first that all drivers are risk neutral. What parking fine would you levy and how many meter monitors would you hire (1, 2, 3, or 4) to achieve the current level of deterrence against illegal parking at the minimum cost?
 - Now assume that drivers are very risk averse. How would your answer to (a) change?
 - (For discussion) What if drivers could insure themselves against the risk of parking fines? Would it make good public policy to allow such insurance to be available?

• Try No. 1, 3, and 6

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 - What would a risk-neutral person pay to play the lottery?

a) Expected value:

$$0.1(100) + 0.2(50) + 0.7(10) = 27$$

b) Variance:

$$0.1(100 - 27)^2 + 0.2(50 - 27)^2 + 0.7(10 - 27)^2 = 841$$

c) \$27

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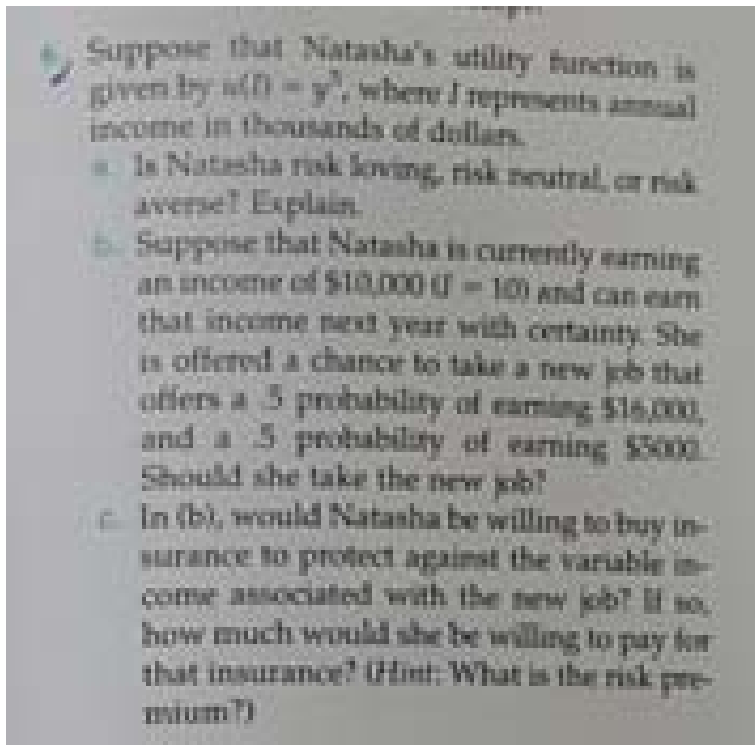
a) Expected value:

$$0.5(0) + 0.25(1) + 0.2(2) + 0.05(7.5) = 1.025$$

Variance:

$$0.5(0-1.025)^2 + 0.25(1-1.025)^2 + 0.2(2-1.025)^2 + 0.05(7.5-1.025)^2 = 2.812$$

- No. He prefers to have \$1 with certainty.
- We might need to draw a picture. Scale everything up 1000 times.
- Up to you



c) Natasha will not buy an insurance.

a) The utility function is $u = y^5$, hence:

$$y = 1 \rightarrow u = 1$$

$$y = 2 \rightarrow u = 32$$

$$y = 3 \rightarrow u = 243$$

This is a convex function, Natasha is a risk lover.

b) Expected utility from the new job is:

$$0.5(16)^5 + 0.5(5)^5 = 525,850.5.$$

The utility from the current job is: $(10)^5 = 100,000$.

She will take the new job.