Instruction:

• Exam time: 120 minutes.
• You may use a calculator, turn off cell phones. Phone communication are strictly prohibited during the exam.
• The exam has 20 pages including the R output. Please check that you have all 20 pages.
• For each question, write your answer in the blank space provided.
• Manage your time carefully and answer as many questions as you can.
• For simplicity, if not specifically given, use 5 % level of significance.
• Round your answer to 3 significant digits.

Question 1 (60 points)

Your score......................

Consider the daily log returns r_t, in percentages, of the Starbucks stock prices (SBUX) for a certain period of time from 3 January 2000 to 27 February 2019 with 4817 observations. Answer the following questions, using the below R output.

Toolbox 1.1

```
> getSymbols('SBUX',from="2000-01-01",to="2019-02-28")
[1] "SBUX"
> head(SBUX)
SBUX.Open SBUX.High SBUX.Low SBUX.Close SBUX.Volume SBUX.Adjusted
2000-01-03  2.984375  3.085938 2.906250   3.082025    24232000      2.471384
2000-01-04  3.007813  3.109375 2.968750   2.984375    21564800      2.393081
2000-01-05  2.992188  3.078125 2.960938   3.023438    28206400      2.424404
2000-01-06  3.000000  3.203125 3.000000   3.132813    30825600      2.512110
2000-01-07  3.093750  3.125000 3.031250   3.117188    26044800      2.499581
2000-01-10  3.234375  3.343750 3.226563   3.250000    29031200      2.606078
> tail(SBUX)
SBUX.Open SBUX.High SBUX.Low SBUX.Close SBUX.Volume SBUX.Adjusted
2019-02-20     70.23     70.60    70.05      70.39     9671800         70.39
2019-02-21     70.14     70.93    70.14      70.70    10933500         70.70
2019-02-22     70.76     71.39    70.75      71.30    11311600         71.30
2019-02-25     71.90     72.07    70.98      71.05    12071200         71.05
2019-02-26     71.00     71.45    70.92      71.14    12302300         71.14
2019-02-27     70.83     71.13    69.92      70.15    13274800         70.15
> chartSeries(SBUX,theme="white")
> xt <- log(as.numeric(SBUX[,6]))
> length(xt)
[1] 4818
> rt <- diff(log(as.numeric(SBUX[,6])))*100
> length(rt)
[1] 4817

> table.Stats(rt)

Observations    4817.0000
NAs                0.0000
Minimum          -20.3629
Quartile 1        -0.9080
Median             0.0303
Arithmetic Mean    0.0695
Quartile 3         1.0135
Maximum           16.8728
SE Mean            0.0306
LCL Mean (0.95)    0.0094
UCL Mean (0.95)    0.1295
Variance           4.5224
Stdev              2.1266
Skewness           0.1526
Kurtosis           7.3810
> jarque.bera.test(rt)

Jarque Bera Test

data:  rt
X-squared = 10953, df = 2, p-value < 2.2e-16

> ### AR(5)
> m1 <- arima(rt,order=c(5,0,0))
> m1

Call:
```

```
arima(x = rt, order = c(5, 0, 0))

Coefficients:
ar1      ar2      ar3      ar4      ar5  intercept
-0.0776  -0.0203  0.0271  -0.0155  -0.0306     0.0694
s.e.   0.0144   0.0144  0.0144   0.0144   0.0144     0.0273

sigma^2 estimated as 4.484:  log likelihood = -10448.96,  aic = 20911.93
> ### Model refinement
> c1 <- c(NA,0,0,0,NA,NA)
> m1a <- arima(rt,order=c(5,0,0),fixed=c1)
Warning message:
In arima(rt, order = c(5, 0, 0), fixed = c1) :
some AR parameters were fixed: setting transform.pars = FALSE
> m1a

Call:
arima(x = rt, order = c(5, 0, 0), fixed = c1)

Coefficients:
ar1  ar2  ar3  ar4      ar5  intercept
-0.0770    0    0    0  -0.0305     0.0694
s.e.   0.0144    0    0    0   0.0144     0.0276

sigma^2 estimated as 4.491:  log likelihood = -10452.67,  aic = 20913.34
> Box.test(m1a$residuals,lag=10,type='Ljung')

Box-Ljung test

data:  m1a$residuals
X-squared = 11.279, df = 10, p-value = 0.3362

> ### Comparison with ARMA(1,2)
> m2 <- arima(rt,order=c(1,0,2))
> m2

Call:
arima(x = rt, order = c(1, 0, 2))

Coefficients:
ar1      ma1      ma2  intercept
-0.4586  0.3821  -0.0569     0.0695
s.e.   0.2539  0.2536   0.0228     0.0277

sigma^2 estimated as 4.491:  log likelihood = -10453.01,  aic = 20900.02
> source("/Users/wasin_siwasarit/Desktop/backtest.R")
> backtest(m1a,rt,orig=4800)
[1] "RMSE of out-of-sample forecasts"
[1] 0.8735982
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.6424781
> backtest(m2,rt,orig=4800)
[1] "RMSE of out-of-sample forecasts"
[1] 0.8684251
[1] "Mean absolute error of out-of-sample forecasts"
[1] 0.641176
```

Question 1.1 (10 points)

Your score.....................

From the Toolbox 1.1, Let μ be the expected value of r_t. Test $H_0 : \mu = 0$ versus $H_a : \mu \geq 0$. Obtain the test statistic and draw your conclusion.

Question 1.2 (10 points)

Your score.....................

Does the distribution of r_t have heavy tails? Write down Null Hypothesis: (H_0) and Alternative Hypothesis: (H_1) to test this. Obtain the test statistic and draw your conclusion.

Question 1.3 (10 points)

Your score......................

Is the distribution of r_t skew? Write down Null Hypothesis: (H_0) and Alternative Hypothesis: (H_1) to test this. Obtain the test statistic and draw your conclusion.

Question 1.4 (5 points)

Your score......................

An AR(5) model is fitted with the refinement. Write down the fitted model after the refinement, including σ^2 of the residuals.

Question 1.5 (5 points)

Your score......................

An ARMA(1,2) model is also estimated to be the alternative model . Write down the fitted model, including σ^2 of the residuals.

Question 1.6 (10 points)

Your score.....................

Is the fitted ARMA(1,2) model adequate? Is it also stationary? Why?

Question 1.7 (10 points)

Your score.....................

Among models [m1a,m2] , which one is preferred? Why?

Question2(50 points) Your score.....................

Given the daily log returns : (R_t) can be explained bt the ARMA (2,3) model as following:

$$(1+0.8B-0.1B^2)R_t = 0.3+(1-0.5B+0.7B^3)\epsilon_t$$

where ϵ_t is distributed as the Gaussian White Noise with mean (μ) = 0 and variance (σ^2) =0.25
B คือ lag-operator

Question2.1 (10 points) Your score.....................

From the above ARMA(2,3) model, Is the model weakly stationary? Write down the reverse characteristic equation and find out the conditions to support your answer.

Question2.2 (10 points)

Your score......................

Calculate the unconditional mean: $E(R_t)$ of R_t

Question2.3 (10 points)

Your score......................

Find out the unconditional variance: $Var(R_t)$ of R_t

Question2.4 (10 points) Your score......................

Calculate the autocorrelation: ρ_l for l=1 and 2 of R_t . Also, write down the autocorrelation: ρ_l when l ≥ 2.

Question2.5 (10 points)

Your score......................

Given $R_{1000} = -0.02$ $R_{999} = 0.05$ $R_{998} = 0.01$ $\epsilon_{1000} = 0.02$ $\epsilon_{999} = 0.04$ $\epsilon_{998} = -0.02$
Obtain 1-step 95 % interval forecasts for R_t at the forecast origin t = 1000.

Question3 (30 points)

Your score.....................

A reseacher from Thammasat University collects the series of (x_t) with 6000 observations. Then, he estimates the 4 alternative models which are m1,m2,m3,m4 as represented in the toolbox 3.1:

Toolbox 3.1

```
> acf(x,lag=80)
> pacf(x,lag.max=80)
> m1 <- arima(x,order=c(1,0,0))
> m1

Call:
arima(x = x, order = c(1, 0, 0))

Coefficients:
ar1  intercept
-0.8966     1.9985
s.e.   0.0050     0.0053

sigma^2 estimated as 0.8065:  log likelihood = -10492.07,  aic =
    20990.14
> Box.test(m1$residuals,lag=10,type='Ljung')

Box-Ljung test

data:  m1$residuals
X-squared = 129.59, df = 10, p-value < 2.2e-16

> m2 <- arima(x,order=c(2,0,0))
> m2

Call:
arima(x = x, order = c(2, 0, 0))

Coefficients:
ar1      ar2  intercept
-0.7858  0.1235     1.9985
s.e.   0.0111  0.0111     0.0060
```

```
sigma^2 estimated as 0.7942:  log likelihood = -10430.53,  aic =
    20869.06
> Box.test(m2$residuals,lag=10,type='Ljung')

Box-Ljung test

data:  m2$residuals
X-squared = 10.167, df = 10, p-value = 0.426

> m3 <- arima(x,order=c(0,0,1))
> m3

Call:
arima(x = x, order = c(0, 0, 1))

Coefficients:
ma1   intercept
-0.7057      1.9985
s.e.    0.0060      0.0046

sigma^2 estimated as 1.92:  log likelihood = -13961.9,  aic = 27929.81
> Box.test(m3$residuals,lag=10,type='Ljung')

Box-Ljung test

data:  m3$residuals
X-squared = 17809, df = 10, p-value < 2.2e-16

> m4 <- arima(x,order=c(0,0,2))
> m4

Call:
arima(x = x, order = c(0, 0, 2))

Coefficients:
ma1      ma2   intercept
-0.8671   0.5026      1.9985
s.e.    0.0106   0.0080      0.0082

sigma^2 estimated as 1.348:  log likelihood = -12545.35,  aic =
    25098.7
```

```
> Box.test(m4$residuals,lag=10,type='Ljung')

Box-Ljung test

data:  m4$residuals
X-squared = 9192, df = 10, p-value < 2.2e-16
```

Figure 3.1 Autocorrelation Function (ACF) of x_t

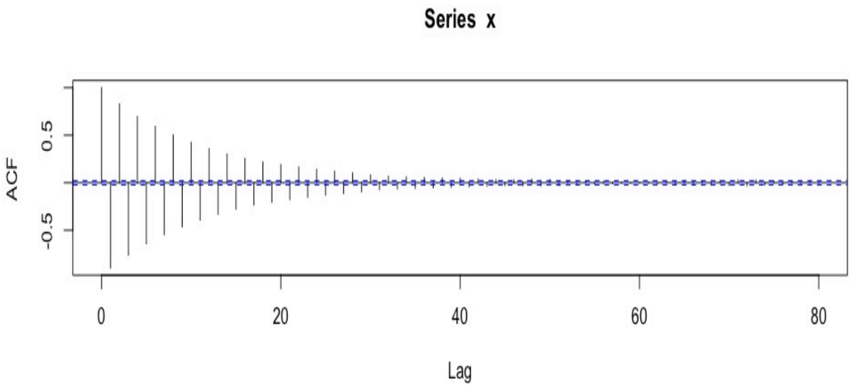
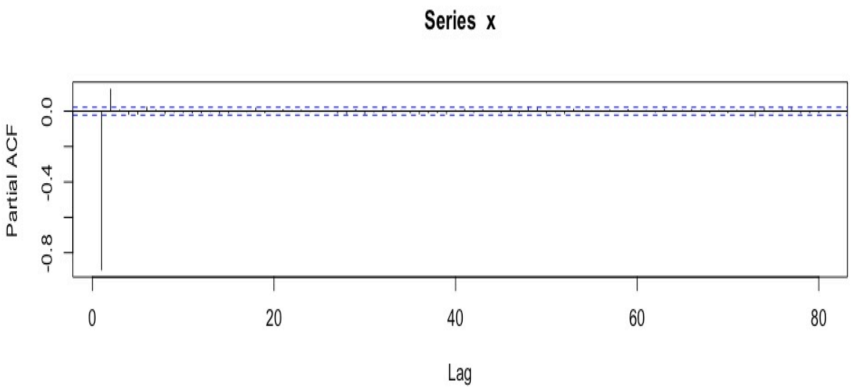


Figure 3.2 Partial Autocorrelation Function (PACF) of x_t



Question3.1 (10 points)

Your score......................

Among models m1, m2, m3, m4, which one is preferred? Why? Write down the fitted model you select, including σ^2 of the residuals.

Question3.2 (10 points)

Your score......................

Calculate the unconditional mean: $E(x_t)$ and the unconditional variance: $Var(x_t)$ based on the selected model on question 3.1.

Question3.3 (10 points)

Your score......................

Calculate the 1-step ahead forecast of x_t :$\hat{x}_h(1)$ and the multi-step ahead forecasts when $l \to \infty$: :$\hat{x}_h(\infty)$. Also conduct the 95 % interval forecasts for the 1-step ahead forecast and the multi-step ahead forecasts when $l \to \infty$.

$a_{6000} = 0.2$ $a_{5999} = 0.4$ $a_{5998} = 0.8$ $a_{5997} = 0.6$

$x_{6000} = 0.1$ $x_{5999} = -0.3$ $a_{5998} = -0.1$ $a_{5997} = -0.2$