

Real options exercise solutions

1. The underlying asset value is the present value of oil reserves if developed now, which is

$$S(0) = \$10 \text{ mn} * (\$20 - \$6) = \$140 \text{ mn}$$

Annual cost of delay is rate of extraction, $\gamma = 4\%$

Exercise price (X) = Cost of developing reserve = \$120 mn

Term to maturity, T = 20 years

Risk-free rate, $r = 7\%$,

STD = 20%

Using the formula,

$$c = S_0 e^{-\gamma T} N(d_1) - X e^{-rT} N(d_2)$$

$$d_1 = 1.2904 \text{ and } d_2 = 0.3960$$

Then using excel function =NORMSDIST(d1) = 0.9015

Then using excel function =NORMSDIST(d2) = 0.6539

Or you can look up standard normal table.

Value is equivalent to the call, c approx = \$ 37.37

2. Value of the underlying asset = Present Value of expected gold sales (@ 50,000 ounces a

$$\text{year}) = (50,000 * 375) * [1 - (1.03^{20}/1.09^{20})]/(.09 - .03) - (50,000 * 250) * [1 -$$

$$(1.05^{20}/1.09^{20})]/(.09 - .05) = \$211.79 \text{ million} - \$164.55 \text{ million} = \$47.24 \text{ million}$$

Note: For computation above, I am using the formula for present value of annuity with growth:

$$\frac{P}{r - g} \left[1 - \left(\frac{1 + g}{1 + r} \right)^n \right]$$

P = First Payment

r = rate per period

g = growth rate

n = number of periods

Exercise price = Cost of opening mine = \$100 million

Variance in ln(gold price) = 0.04

Time to expiration on the option = 20 years

Riskless interest rate = 6%

Dividend Yield = Loss in production for each year of delay = 1 / 20 = 5%

(Note: It will take 20 years to empty the mine, and the firm owns the rights for 20 years.

Every year of delay implies a loss of one year of production.)

Based upon these inputs, the Black-Scholes model provides the following value for the call: $d1 =$

-0.1676 $N(d1) = 0.4334$

$d2 = -1.0621$ $N(d2) = 0.1441$

Call Value = $47.24 \exp(-0.05)(20) (0.4334) - 100 (\exp(-0.09)(20) (0.1441)) = \$ 3.19$ million.

The value of the mine as an option is \$ 3.19 million, in contrast the static capital budgeting analysis would have yielded a net present value of -\$52.76 million (\$47.24 million - \$100 million). The additional value accrues directly from the mine's option characteristics.

3. a) $S(0) = \text{PV of CF from development} = 900 * 0.4 = \360 mn

Annual cost of abandonment delay, $Y = 1/5$

Exercise price = Abandonment value = \$300 mn

$T = 5$ years

$R = 5\%$

$\text{STD} = 40\%$

$$p = Xe^{-rT} N(-d_2) + S_0 e^{-YT} N(-d_1)$$

p approx 125

b) Conventional NPV = -400+360 = \$ -40 mn

NPV with option value to sell= Conventional NPV + Option value = - 40mn + 125 mn = -85 mn

Since value of NPV with option is greater than 0, the firm should invest in the project.

4. Without option to expand NPV is -20 mn.

$$S(0) = 100$$

Value of additional investment = Exercise price = 150

$$R = 6.5\%$$

$$T = 10 \text{ years}$$

$$STD = 0.1^{0.5} = 0.316$$

$$c = 100 \cdot N(0.744535) - 150e^{-0.065 \cdot 10} N(-0.25547)$$

$$c = 100 \cdot (0.7717) - 150e^{-0.65} (0.3992) = 45.9$$

NPV with option = Conventional NPV (Existing investment) + option value to expand project =
-20 + 45.9 = 25.9.

Hence, the firm should undertake the project.

c) When using option concept to value a company, the exercise value is the duration weighted value of debt = 8865 mn

Note: Duration is the average maturity of debt, weighted by the times when payments are made. If this information is unavailable, then using the outstanding average maturity of debt will suffice. (ie. Replace duration column in table with maturity of debt in years).

I have not covered the details of computing duration in class so you will not be asked to compute for it in the exam, however, it is a very important to at least know how to interpret it.

d) Call value of the firm's equity = 122 mn

Using inputs

$$S(0) = 2,312$$

$$X = 8,865$$

$$T = 10.93$$

$$STD = 0.035^{0.5} = 18.3\%$$