

### Solution: Quiz 3

1. Consider a function

$$f(x) = 4x - x^4.$$

- (a) Find the increasing and decreasing intervals.
- (b) Find the relative maximum and minimum.
- (c) Find the concavity and inflection point.
- (d) Sketch the graph of  $f(x)$ .

Note:  $f(x) = 4x - x^4$ ,  $f'(x) = 4 - 4x^3$ ,  $f''(x) = -12x^2$

**Solution:**

(a) and (b)

$$f'(x) = 4 - 4x^3 = 4(1 - x^3) = 0 \Rightarrow x^3 = 1 \Rightarrow$$

$x = 1$  are the only critical numbers.

	$x \in (-\infty, 1)$	$x \in (1, \infty)$
Sign of $f'(x) = 4(1 - x^3)$	+	-

- From the table in (iii), we see that  $f(x)$  is increasing ( $f'(x) > 0$ ) on  $(-\infty, 1)$  and  $f(x)$  is decreasing ( $f'(x) < 0$ ) on  $(1, \infty)$ .

- The First Derivative Test:

$f(1) = 4(1) - 1^4 = 3$  is a relative maximum (since the sign of  $f'(x)$  changes from (+) to(-) ).

(c) To find the inflection points, we first solve  $f''(x) = 0$  and see the sign change of  $f''(x)$ :

$$f''(x) = -12x^2 = 0 \implies x = 0.$$

	$x \in (-\infty, 0)$	$x \in (0, \infty)$
Sign of $f''(x) = -12x^2$	-	-

Since the sign of  $f''(x)$  doesn't change at  $x = 0$ , there is **no inflection point**.

From the table above, we see that

$f(x)$  is concave down ( $f''(x) < 0$ ) on  $(-\infty, 0) \cup (0, \infty)$ .

(d)

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