EE431/438 Economics of Financial Markets and Institutions

Exercise 2: Decision Under Uncertainty

1. Ms. Mary Kelly has initial wealth $W_0 = \$1,200$ and faces an uncertain future that she partitions into two states, $s = 1$ and $s = 2$. She can invest in two securities, j and k, with initial prices of $p_j = \$10$ and $p_k = \$12$, and the following payoff table:

Security	s = 1	s = 2	Security Price
j	\$10	\$12	\$10
k	\$20	\$8	\$12

   (a) If she buys only security j, how many shares can she buy? If she buys only security k, how many shares can she buy? What would her final wealth, W_s, be in both cases and each state?

   If Ms.Mary Kelly buys only security j, the amount of shares she can buy is equal to $\dfrac{\text{total wealth}}{\text{the price of security} j} = \dfrac{1200}{10} = 120\,\text{shares}$.
   Her wealth would in each state becomes $W_1 = 1200$ and $4W_2 = 1440$.

   If she buys only security k, the amount of shares she can buy is equal to $\dfrac{\text{total wealth}}{\text{the price of security} k} = \dfrac{1200}{12} = 100\,\text{shares}$.
   Her wealth would in each state becomes $W_1 = 2000$ and $W_2 = 800$.

   (b) What are the prices of the pure securities implicit in the payoff table?

   Given that the price of securities are fair, price of a security = its expected return. Let π_1 denotes the price for the pure security for state 1 and π_2 denotes the price for the pure security for state 2.

   $10 = 10\pi_1 + 12\pi_2$
   $12 = 20\pi_1 + 8\pi_2$

   Then, $\pi_1 = \dfrac{2}{5}, \pi_2 = \dfrac{1}{2}$.

   (c) What is the initial price of a third security i, for which the payoff in state 1 is \$5 and the payoff in state 2 is \$12.

   The fair price of the third security is equal to its expected return. Thus, the fair price of the third security is : $\left(5 \times \dfrac{2}{5}\right) + \left(12 \times \dfrac{1}{2}\right) =$

$2 + 6 = 8$

(d) If she want her wealth in both states to be equal, how many security j and security k she should buy ? (she is allowed to buy fractions of shares.)

Given that if she buys j units of security j and k units of security k, her wealth in both states would be equal. Then we can write:

$$\begin{array}{ccc} \text{her wealth in state 1} & = & \text{her wealth in state 2} \\ 10j + 20k & = & 12j + 8k \\ \frac{k}{j} & = & \frac{1}{6} \end{array}$$

If she has \$7, she should buy 1 unit of secuity k and 6 units of security j in order that her wealth in both states will be equal. She has \$1,200. Therefore, she should buy $\frac{1200}{7}$ units of security k and $\frac{6}{7}(1200)$ units of security j.

* **Note** : $\pi_1 \neq 1 - \pi_2$. **This is because there could be more than 2 states of the world. The information on the two securities given is sufficient to find the prices of the two pure securities for the two states of nature.**

2. Ms.Brenda only cares about the level of her wealth. Her preferences are represented by the following utility function: $U = \sqrt{W}$, where W is wealth. Her current wealth is £400. Ms.Brenda considers whether or not to buy corporate bonds issued by the Trustmeplease Company. One bond costs £200 pounds. With probability p = 0.5, the bond will pay back £329 (i.e. a net gain of £129). With probability 1 − p = 0.5, the bond will only pay back £161 (i.e. a net loss of £39).

   (a) Should Ms.Brenda invest in one of these bonds? (Hints: compare her utility in absence of any investment with her utility if he buys one bond.)

   Ms. Brenda's utility in absence of any investment is $U = \sqrt{400} = 20$.

   If Ms. Brenda purchases one bond, her expected utility is:

   $EU = 0.5\sqrt{200 + 329} + 0.5\sqrt{200 + 161} = 0.5\sqrt{529} + 0.5\sqrt{361} = 21.$

   $EU = 0.5\sqrt{400 - \text{price} + \text{payoff}} + 0.5\sqrt{400 - \text{price} + \text{payoff}}$

   This is larger than Ms.Brenda's utility in absence of any investment. Therefore, Ms. Brenda should invest in one of the bonds.

   (b) Should Ms.Brenda buy two of these bonds? (Hints: compare her utility in absence of any investment with her utility if he buys two bonds.)

If Ms. Brenda purchases one bond, her expected utility becomes:

$EU = 0.5\sqrt{2(329)} + 0.5\sqrt{2(161)} = 0.5\sqrt{529} + 0.5\sqrt{322} = 21.80.$

This is larger than Ms.Brenda's utility in absence of any investment and it is also larger than her utility from buying one of the bonds. Therefore, Ms. Brenda should indeed invest in two of the bonds.

(c) What is the maximum price P that Ms. Brenda would pay for one bond yielding £284 (i.e. net gain of £284) with a probability of 0.5 and £124 with a probability of 0.5? [Hints: At the maximum price, her utility in absence of any investment his utility if she buys one of the bond at price P. Just write down the formula that would determine P. If you want to solve it, plug in values to get an approximation.]

The question is wrongly set. We cannot calculate the price given this information.

The question must be changed to the following.

What is the maximum price P that Ms. Brenda would pay for one bond which pays back £284 with a probability of 0.5 and £124 with a probability of 0.5? [Hints: At the maximum price, her utility in absence of any investment his utility if she buys one of the bond at price P. Just write down the formula that would determine P. If you want to solve it, plug in values to get an approximation.]

Then, this case we can calculate the maximum price she would buy. Ms. Brenda will purchase the bond as long as her expected utility remains higher than 20, which is his expected utility if he does not purchase the bond. This means that the maximum price, B, that Ms. Brenda is willing to pay for the bond is defined by the following equation:

$0.5\sqrt{400 - P + \text{payoff}} + 0.5\sqrt{400 - P + \text{payoff}} = \sqrt{400}$

$0.5\sqrt{684 - P} + 0.5\sqrt{524 - P} = 20$

The maximum price that Mr.Goldbar is willing to pay is 200.

NOTE: I uploaded an additional sheet for topic 2 today. Please also have a look at be.moodle for topic 2, under the topic, additional supplemenary topic 2. Its content is exactly the same as the lecture. Nothing is new. It is for a brief review.