Solutions to Homework Assignment #4

1. What is the difference between the expected value of a lottery and the expected utility of a lottery?

The expected value of a lottery measures the average payoff of the lottery in monetary terms. The expected utility of a lottery takes into account how much the decision maker values the expected payoff, particularly in terms of the risk associated with the expected payoff. For example, while the expected payoff from one lottery may exceed the expected payoff from a second lottery, if the second lottery has less risk associated with it, a decision maker might prefer the second lottery to the first.

11. You have a utility function given by $U = 2I + 10\sqrt{I}$. You are considering two job opportunities. The first pays a salary of $40,000 for sure. The other pays a base salary of $20,000, but offers the possibility of a $40,000 bonus on top of your base salary. You believe that there is a 0.50 probability that you will earn the bonus.
a) What is the expected salary under each offer?
b) Which offer gives you the higher expected utility?
c) Based on your answer to (a) and (b), are you risk averse?

a) The expected salary under the first job offer is $40,000. The expected salary under the second job offer is also $40,000: 0.5($20,000) + 0.5($60,000) = $40,000.
b) Your expected utility under the first offer is

$$U = 2(40000) + 10\sqrt{40000} = 82{,}000$$

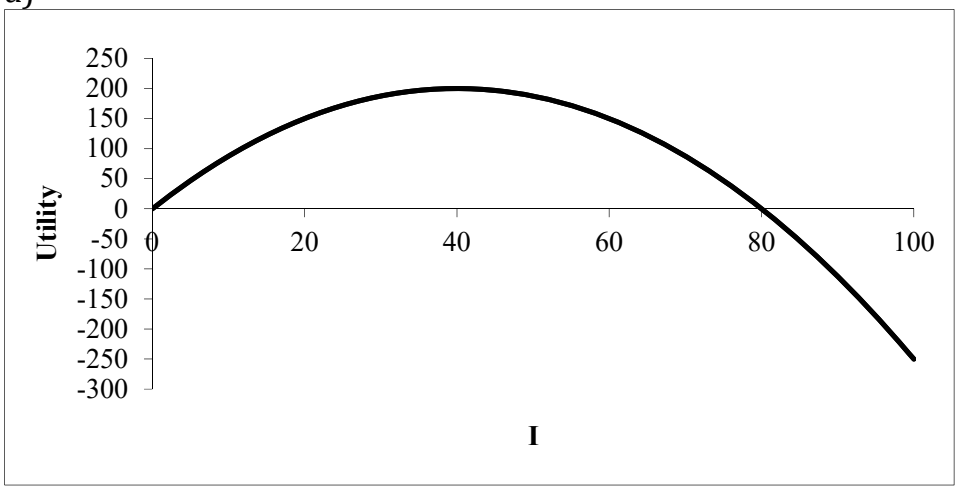
Your expected utility under the second offer is

$$U = .5\left(2(20000) + 10\sqrt{20000}\right) + .5\left(2(60000) + 10\sqrt{60000}\right) = 81{,}932.$$

The first offer gives you the higher expected utility.
c) The two offers have the same expected value. Since you prefer the certain salary to the risky salary, it follows that you are risk averse.

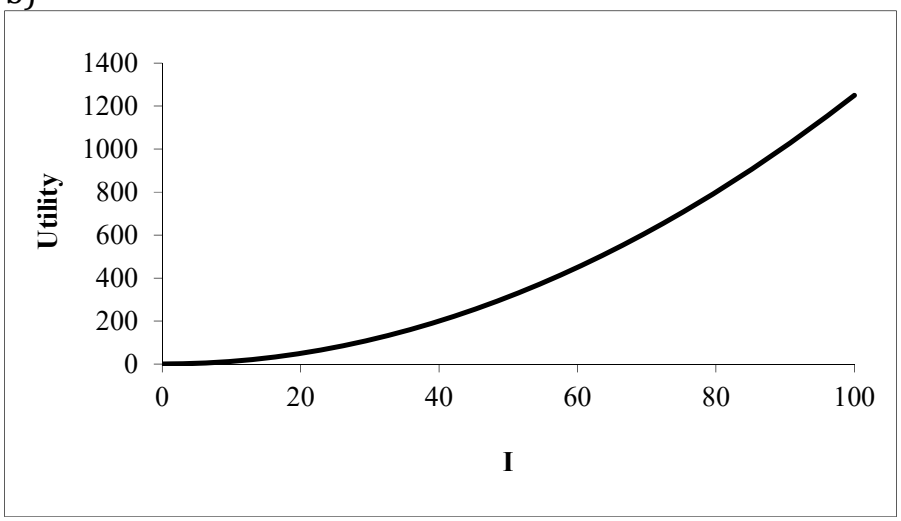
13. Sketch the graphs of the following utility functions as *I* varies over the range $0 to $100. Based on these graphs, indicate whether the decision maker is risk averse, risk neutral, or risk loving:
a) $U = 10I - (1/8)I^2$
b) $U = (1/8)I^2$
c) $U = \ln(I + 1)$
d) $U = 5I$

a)



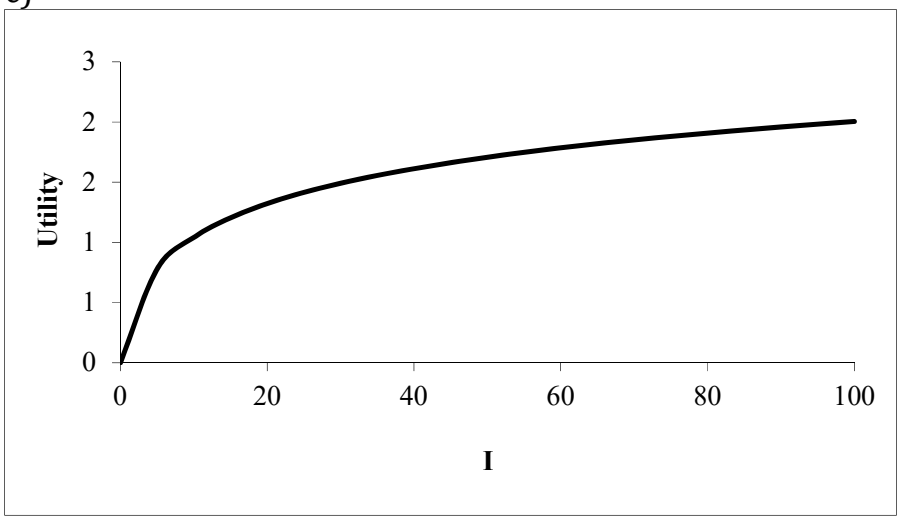
Since this utility function increases at a decreasing rate, the decision maker will prefer a sure thing to a lottery with the same expected value. Thus, this utility function corresponds to a risk-averse decision maker.

b)



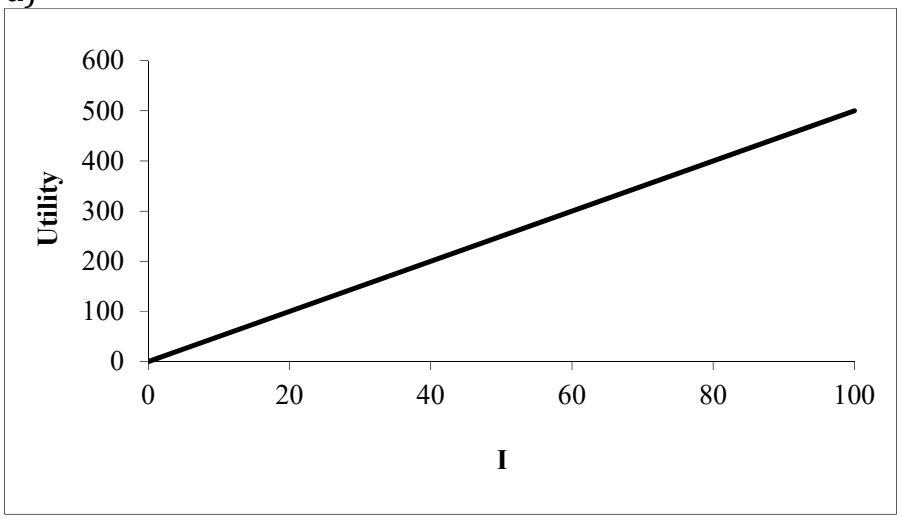
Since this utility function increases at an increasing rate, the decision maker will prefer a lottery to a sure thing with the same payoff. Thus, this utility function corresponds to a risk-loving decision maker.

c)



Since this utility function increases at a decreasing rate, the decision maker will prefer a sure thing to a lottery with the same expected value. Thus, this utility function corresponds to a risk-averse decision maker.

d)



Since this utility function increases at a constant rate, the decision maker will be indifferent between a sure thing and a lottery with the same expected value. Thus, this utility function corresponds to a risk-neutral decision maker.

14. Suppose that your utility function is U = √ I. Compute the risk premium of the lottery where there is a 0.90 chance that you receive a payoff of $0 and a 0.10 chance that you receive a payoff of $400.

If your utility function were $U = \sqrt{I}$, then the risk premium associated with Lottery *A* would be

$$0.90\sqrt{0} + 0.10\sqrt{400} = \sqrt{40 - RP_A}$$
$$\sqrt{40 - RP_A} = 2$$
$$40 - RP_A = 4$$
$$RP_A = 36$$

17. Consider a household that possesses $100,000 worth of valuables (computers, stereo equipment, jewelry, and so forth). This household faces a 0.10 probability of a burglary. If a burglary were to occur, the household would have to spend $20,000 to replace the stolen items. Suppose it can buy an insurance policy for $500 that would fully reimburse it for the amount of the loss.
a) Should the household buy this insurance policy?
c) What is the most the household would be willing to pay for this insurance policy? How does your answer relate to the concept of risk premium discussed in the text?

a) If you remain uninsured, you face a lottery in which you have 10% chance of $80,000 in valuables and a 90% chance of $100,000 in valuables. The expected value of valuables is thus $98,000.
If you purchase the insurance policy for $500, then with no burglary you have $100,000 - 500 = \$99,500$ and with a burglary you have $100,000 - 500 - 20,000 + 20,000 = \$99,500$. The expected value if you purchase the policy is therefore $99,500. Since the expected value at year end with insurance exceeds the expected value at year end without insurance, you should purchase the insurance policy for $500.

c) Since without the policy you would have an expected value $2,000 less than the value of the valuables, the most you would be willing to pay for an insurance policy that fully reimburses for loss is $2,000.

19. You are a relatively safe driver. The probability that you will have an accident is only 1 percent. If you do have an accident, the cost of repairs and alternative transportation would reduce your disposable income from $120,000 to $60,000. Auto collision insurance that will fully insure you against your loss is being sold at a price of $0.10 for every $1 of coverage. Finally, suppose that your utility function is $U = \sqrt{I}$.
You are considering two alternatives: buying a policy with a $1,000 deductible that essentially provides just $59,000 worth of coverage, or buying a policy that fully insures you against damage. The price of the first policy is $5,900. The price of the second policy is $6,000. Which policy do you prefer?

Your expected utility if you buy the first policy is

$$0.01\sqrt{120{,}000-60{,}000+59{,}000-5900}+0.99\sqrt{120{,}000-5900}=337.77.$$

If you buy the second policy, your expected utility is

$$0.01\sqrt{120{,}000-60{,}000+60{,}000-6000}+0.99\sqrt{120{,}000-6000}=337.64$$

Your expected utility is higher when you buy the first policy.

20. Consider a market of risk-averse decision makers, each with a utility function $U = \sqrt{I}$. Each decision maker has an income of $90,000, but faces the possibility of a catastrophic loss of $50,000 in income. Each decision maker can purchase an insurance policy that fully compensates her for her loss. This insurance policy has a cost of $5,900. Suppose each decision maker potentially has a different probability q of experiencing the loss.
a) What is the smallest value of q so that a decision maker purchases insurance?

a) If an individual purchases insurance, her (certain) utility is

$$\sqrt{90{,}000-5{,}900}=290.$$

If an individual does not purchase insurance, her expected utility is

$$q\sqrt{90{,}000-50{,}000}+(1-q)\sqrt{90{,}000}$$
$$=200q+300(1-q)=300-100q$$

An individual will purchase insurance if

$$290\geq 300-100q, \text{ or}$$
$$q\geq 0.10.$$

In other words, individuals that are 90 percent or more certain that they will not experience the loss will not purchase insurance.

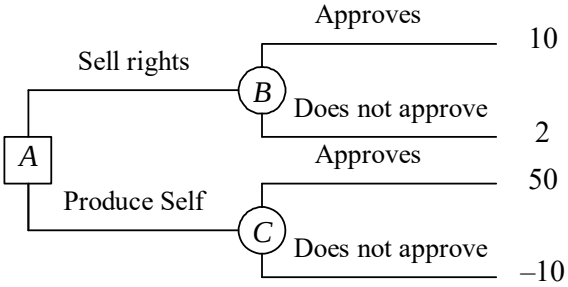
22. A small biotechnology company has developed a burn treatment that has commercial potential. The company has to decide whether to produce the new compound itself or sell the rights to the compound to a large drug company. The payoffs from each of these courses of action depend on whether the treatment is approved by the Food and Drug Administration (FDA), the regulatory body in the United States that approves all new drug treatments. (The FDA bases its decision on the outcome of tests of the drug's effectiveness on human subjects.) The company must make its decision before the FDA decides. Here are the payoffs the drug company can expect to get under the two options it faces:

		Decision	
Outcome	Probability	Sell the Rights	Produce Yourself
FDA approves	0.20	$10	$50
FDA does not approve	0.80	$ 2	−$10

(payoffs are in millions of dollars)

a) Draw a decision tree showing the decisions that the company can make and the payoffs from following those decisions. Carefully distinguish between chance nodes and decision nodes in the tree.
b) Assuming that the biotechnology company acts as a risk-neutral decision maker, what action should it choose? What is the expected payoff associated with this action?
c) Suppose that the biotech company can conduct its own test—at no cost—that will reveal whether the new drug will be approved by the FDA. What is the biotech company's VPI?

a) *A* is a decision node; *B* and *C* are chance nodes.



b) The expected payoff for "Sell rights" is 0.20(10) + 0.80(2) = 3.60. The expected payoff for "Produce yourself" is 0.20(50) + 0.80(–10) = 2.0. Therefore, the risk-neutral company should sell the rights, for an expected payoff of 3.60.

c) If the test indicates the FDA will approve, the company will choose "Produce yourself" and earn a payoff of 50. If the test indicates the FDA will not approve, then the company will choose "Sell the rights" and earn 2. The expected payoff from conducting the costless test is therefore 0.20(50) + 0.80(2) = 11.60.
The VPI is the difference between the expected payoff with the test and the expected payoff without the test. Thus, VPI = 11.60 – 3.60 = 8.00.