FN452: Asset management and portfolio analysis

Portfolios with more than two assets

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- *n* risky assets + risk-free asset
- Practical issues in modern portfolio theory

Portfolios with more than two assets

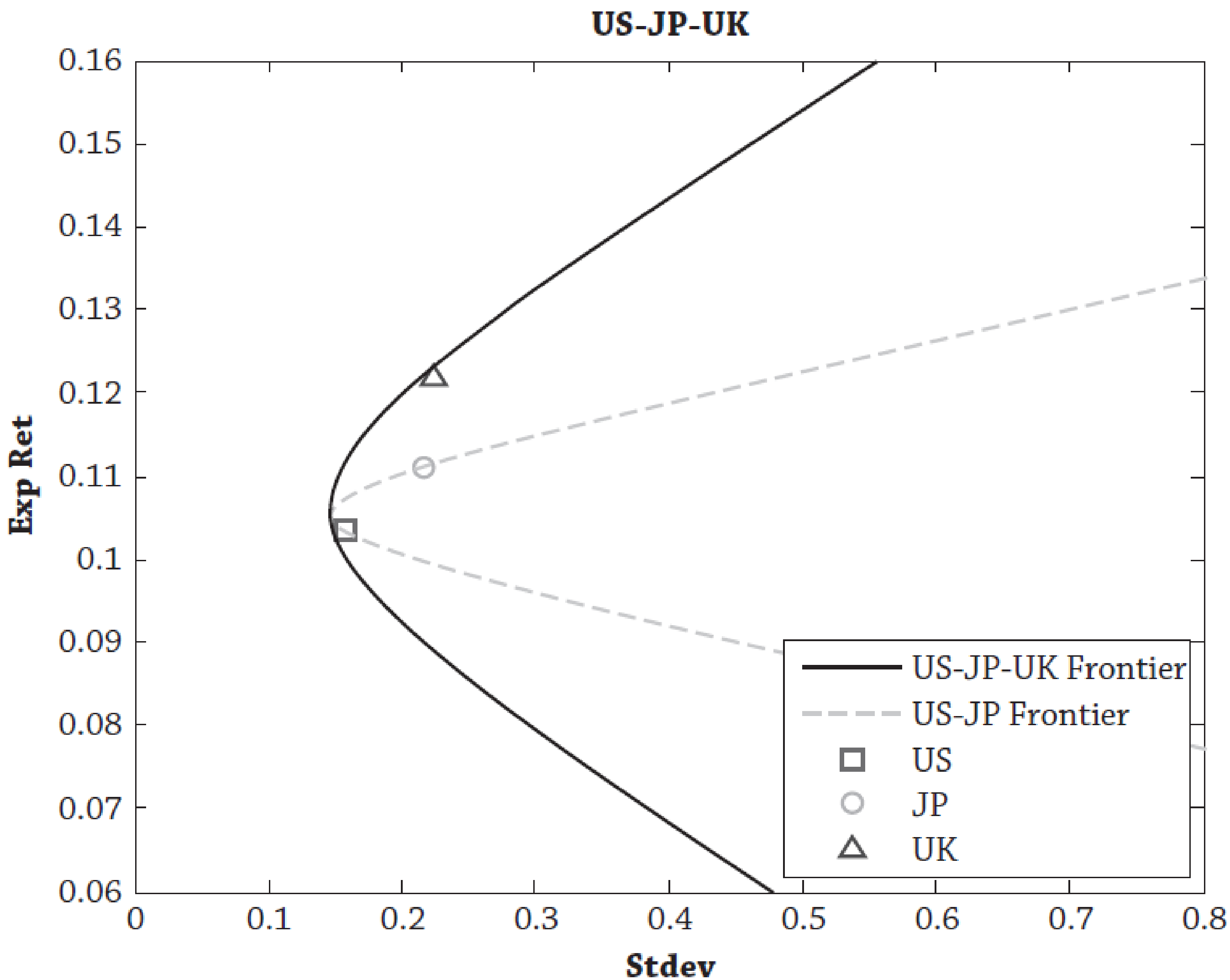
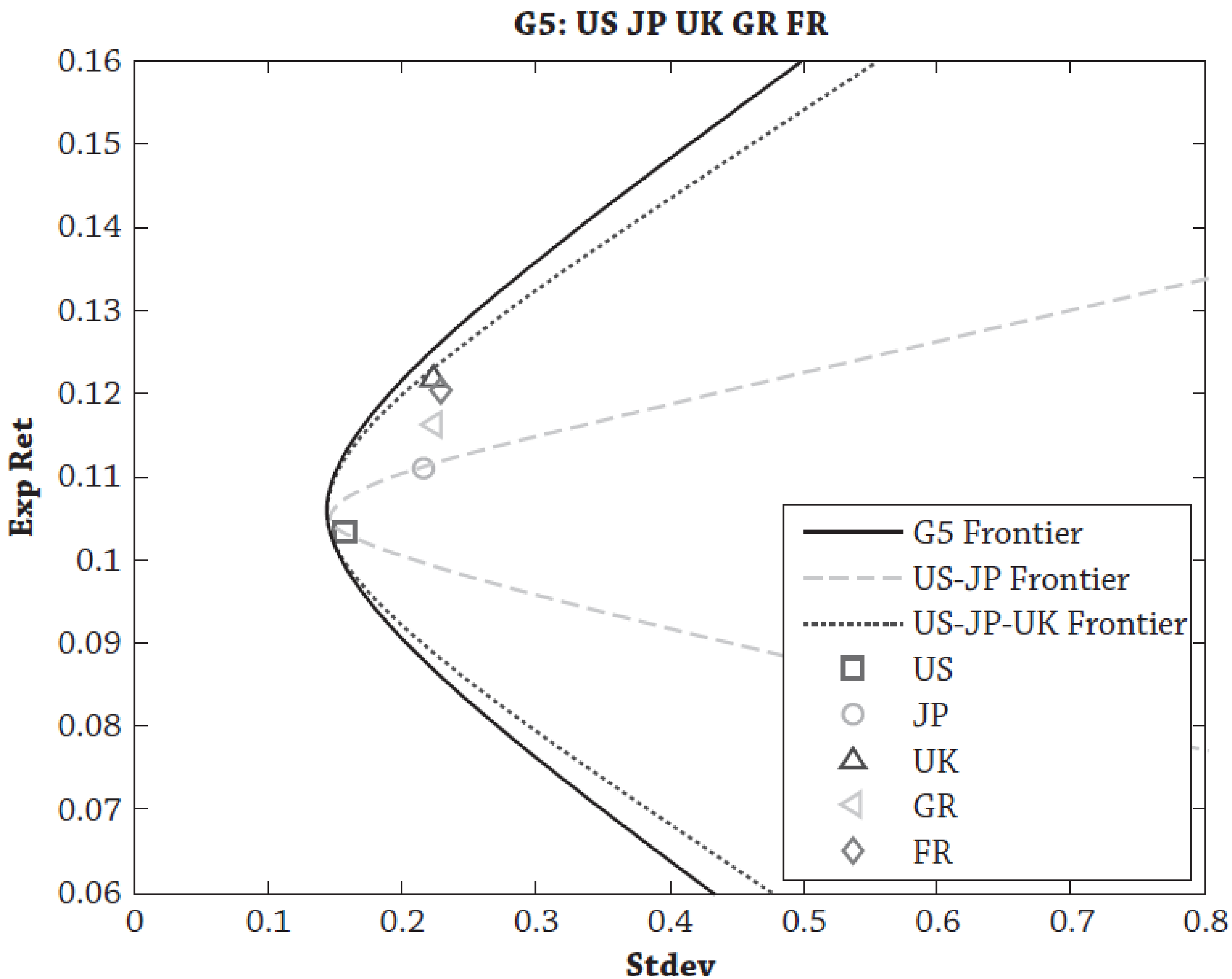


Figure 3.4

Source: Ang (2014)

Portfolios with more than two assets



Source: Ang (2014)

Figure 3.5

n risky assets + risk-free asset

- Expected return of complete portfolio

$$E(R_c) = yE(R_p) + (1 - y)R_f$$

$$E(R_c) = R_f + y[E(R_p) - R_f]$$

- Risk of complete portfolio

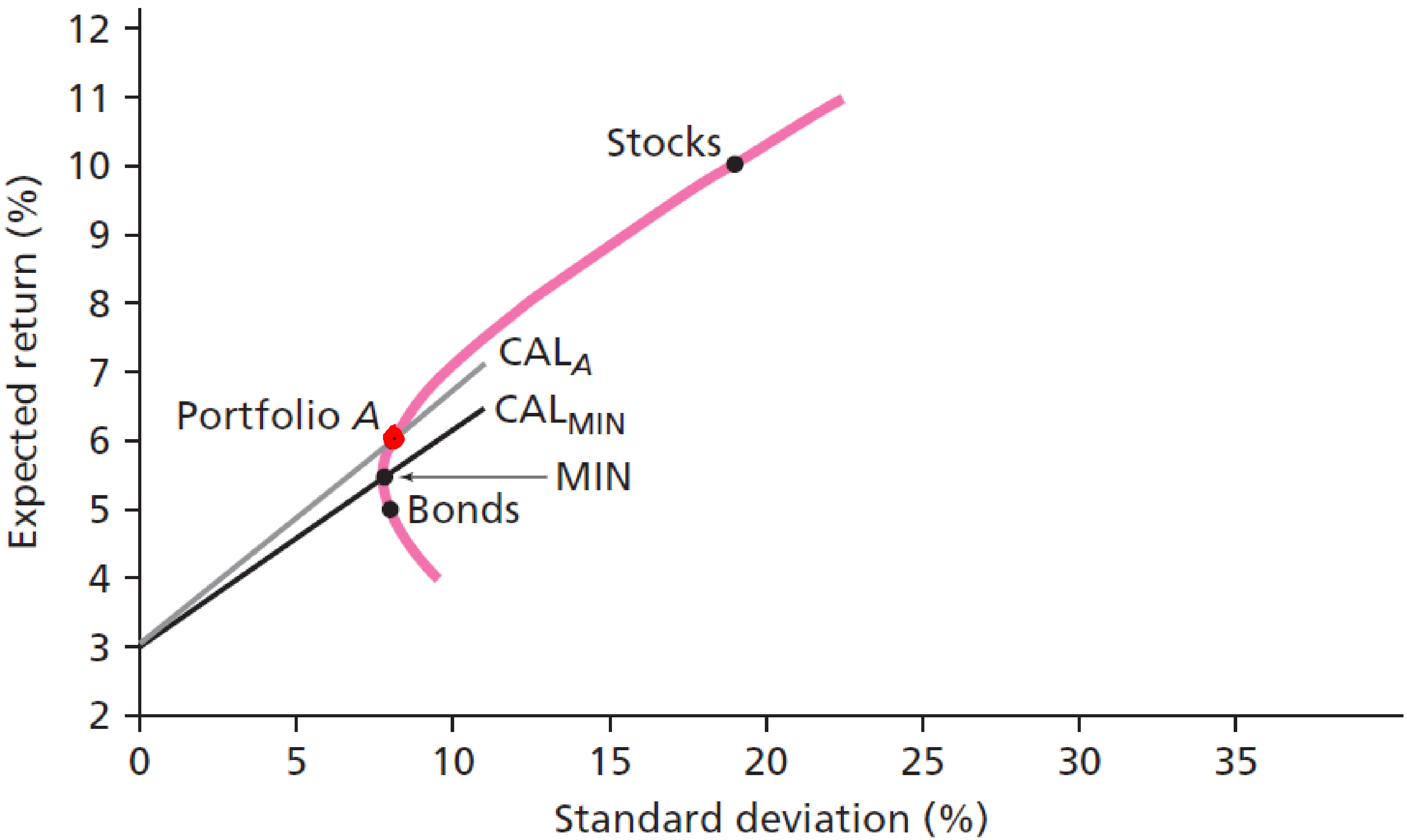
$$\sigma_c = y\sigma_p$$

- Substituting $y = \frac{\sigma_c}{\sigma_p}$ into the expected return equation, we obtain

$$E(R_c) = R_f + \sigma_c \left[\frac{E(R_p) - R_f}{\sigma_p}\right]$$

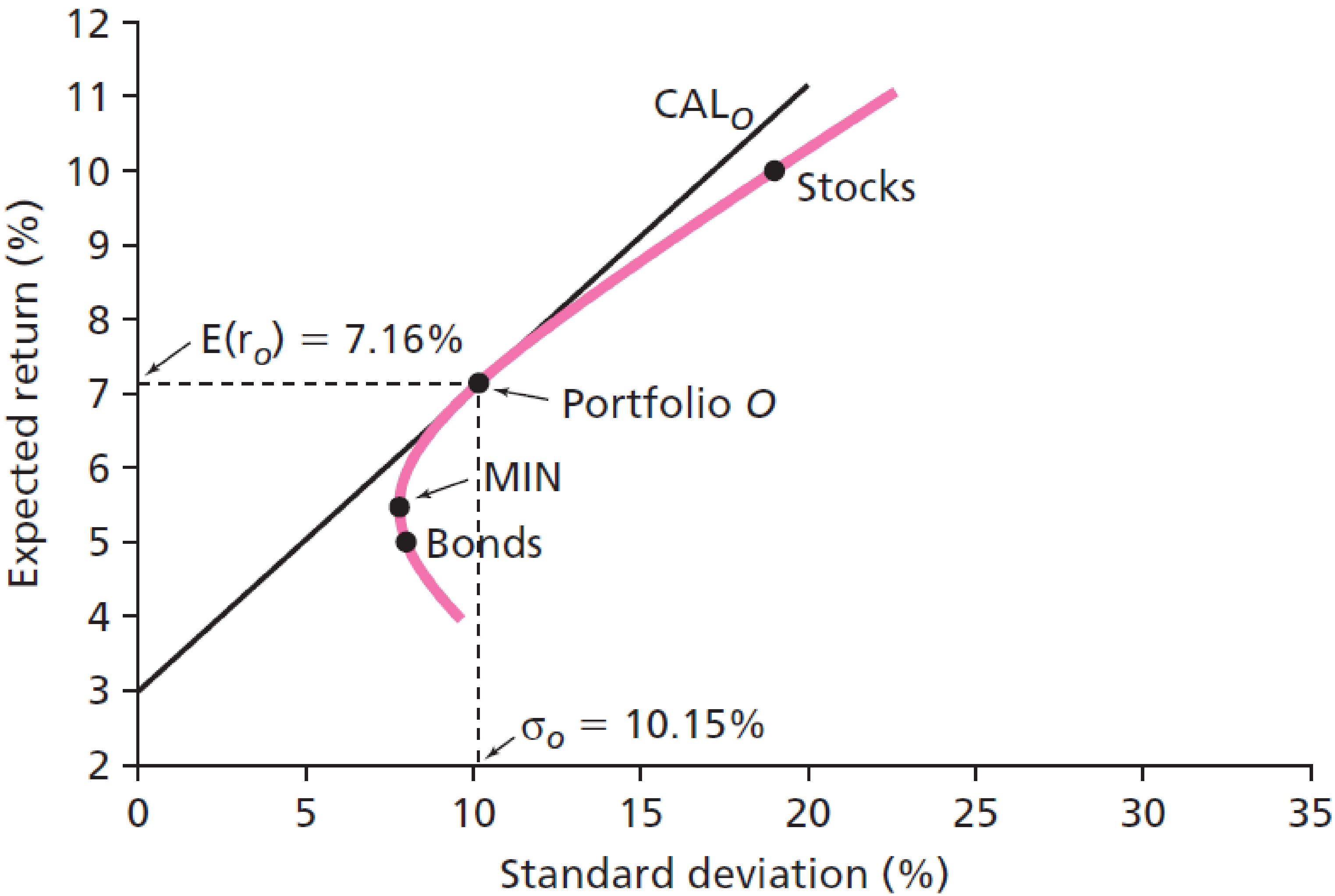
- This is called "Capital Allocation Line (CAL)" - plot of risk-return combinations available by varying allocation between risky and risk-free

n risky assets + risk-free asset



Source: Bodie, Kane, and Marcus (2021)

n risky assets + risk-free asset



Source: Bodie, Kane, and Marcus (2021)

n risky assets + risk-free asset

- Optimal Risky Portfolio
  - Best combination of risky and safe assets to form portfolio
  - To maximize the Sharpe ratio (wrt w) and set it equal to zero
  - Solve for the following formula

$$w_B = \frac{[E(r_B) - r_f]\sigma_S^2 - [E(r_s) - r_f]\sigma_B \sigma_S \rho_{BS}}{[E(r_B) - r_f]\sigma_S^2 + [E(r_s) - r_f]\sigma_B^2 - [E(r_B) - r_f + E(r_s) - r_f]\sigma_B \sigma_S \rho_{BS}}$$

n risky assets + risk-free asset

- Substituting the following parameters into optimal weights (with risk-free rate = 3%), we obtain

$E(r_S)$	$E(r_B)$	σ_S	σ_B	ρ_{BS}
10	5	19	8	0.2

$$w_B = 56.8\% \qquad w_S = 43.2\%$$

- And we can use the following formula to compute for the risky portfolio weight

$$y = \frac{E(r_p) - r_f}{A\sigma_p^2}$$

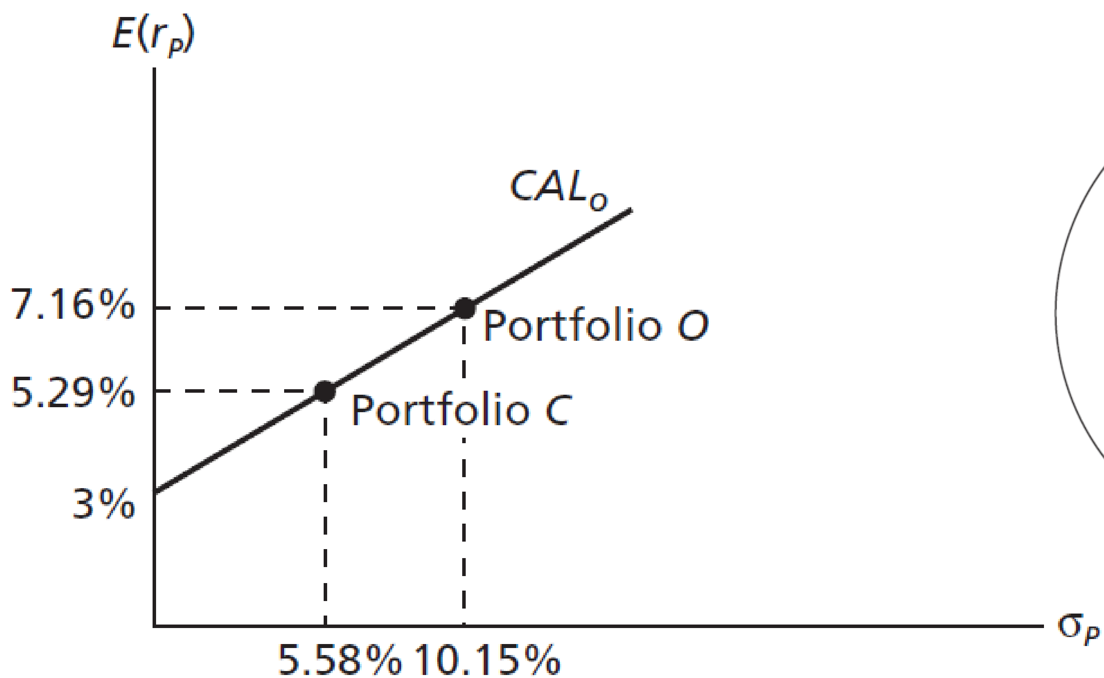
- Then we can calculate for expected return and risk of the complete portfolio as follows. $E(r_C) = 3\% + 55\%(7.16\% - 3\%) = 5.29\%$

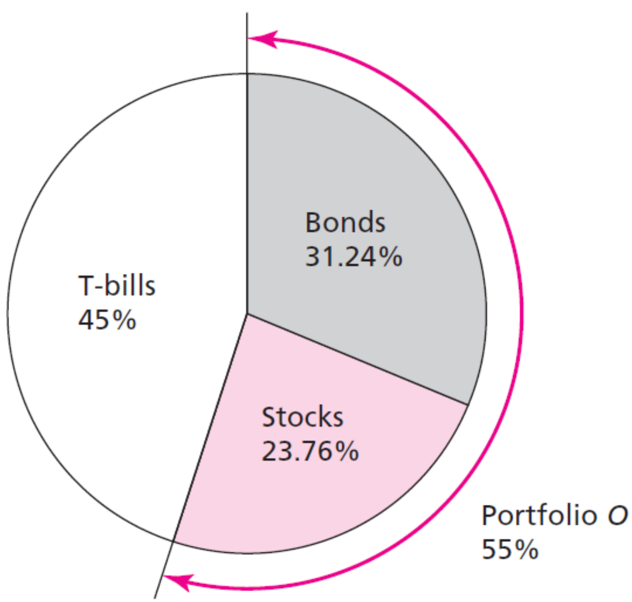
$$\sigma_C = 55\% \times 10.15 = 5.58\%$$

n risky assets + risk-free asset

- Therefore, the overall asset allocation of the complete portfolio is as follows:

Weight in risk-free asset		45.00%
Weight in bond fund	.568 × 55% =	31.24
Weight in stock fund	.432 × 55% =	23.76
Total		100.00%





Source: Bodie, Kane, and Marcus (2021)

n risky assets + risk-free asset

- 3 risky assets

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3)$$

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2$$
$$+2w_1w_2\sigma_{1,2} + 2w_1w_3\sigma_{1,3} + 2w_2w_3\sigma_{2,3}$$

- n risky assets

$$E[R_p] = E\left[\sum_i w_i R_i\right] = \sum_i E[w_i R_i] = \sum_i w_i E[R_i]$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{i,j}$$

n risky assets + risk-free asset

- We can write relevant variables in matrix form as follows

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad \mu = E(r) = \begin{bmatrix} E(R_1) \\ E(R_2) \\ \vdots \\ E(R_n) \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{12} & \cdots & \sigma_1\sigma_n\rho_{1n} \\ \sigma_1\sigma_2\rho_{12} & \sigma_2^2 & \cdots & \sigma_2\sigma_n\rho_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \sigma_1\sigma_n\rho_{1n} & \sigma_2\sigma_n\rho_{2n} & \cdots & \sigma_n^2 \end{bmatrix}$$

Expected return = $\boldsymbol{w}^{\mathrm{T}}\mu$ and risk = $\left(\boldsymbol{w}^{\mathrm{T}}\Sigma\boldsymbol{w}\right)^{1/2}$

Investors maximize the Sharpe ratio

$$\max \frac{\boldsymbol{w}^{\mathrm{T}}\mu - R_f}{(\boldsymbol{w}^{\mathrm{T}}\Sigma\boldsymbol{w})^{1/2}} \quad \text{subject to} \quad \sum w_i = w_1 + w_2 + \cdots + w_n = 1$$

To receive the optimal risky portfolio (or tangency portfolio)

$$\boldsymbol{w}_{tan} = \frac{\Sigma^{-1}\left(\mu - R_f\mathbf{1}\right)}{\mathbf{1}^{\mathrm{T}}\Sigma^{-1}\left(\mu - R_f\mathbf{1}\right)}$$

n risky assets + risk-free asset

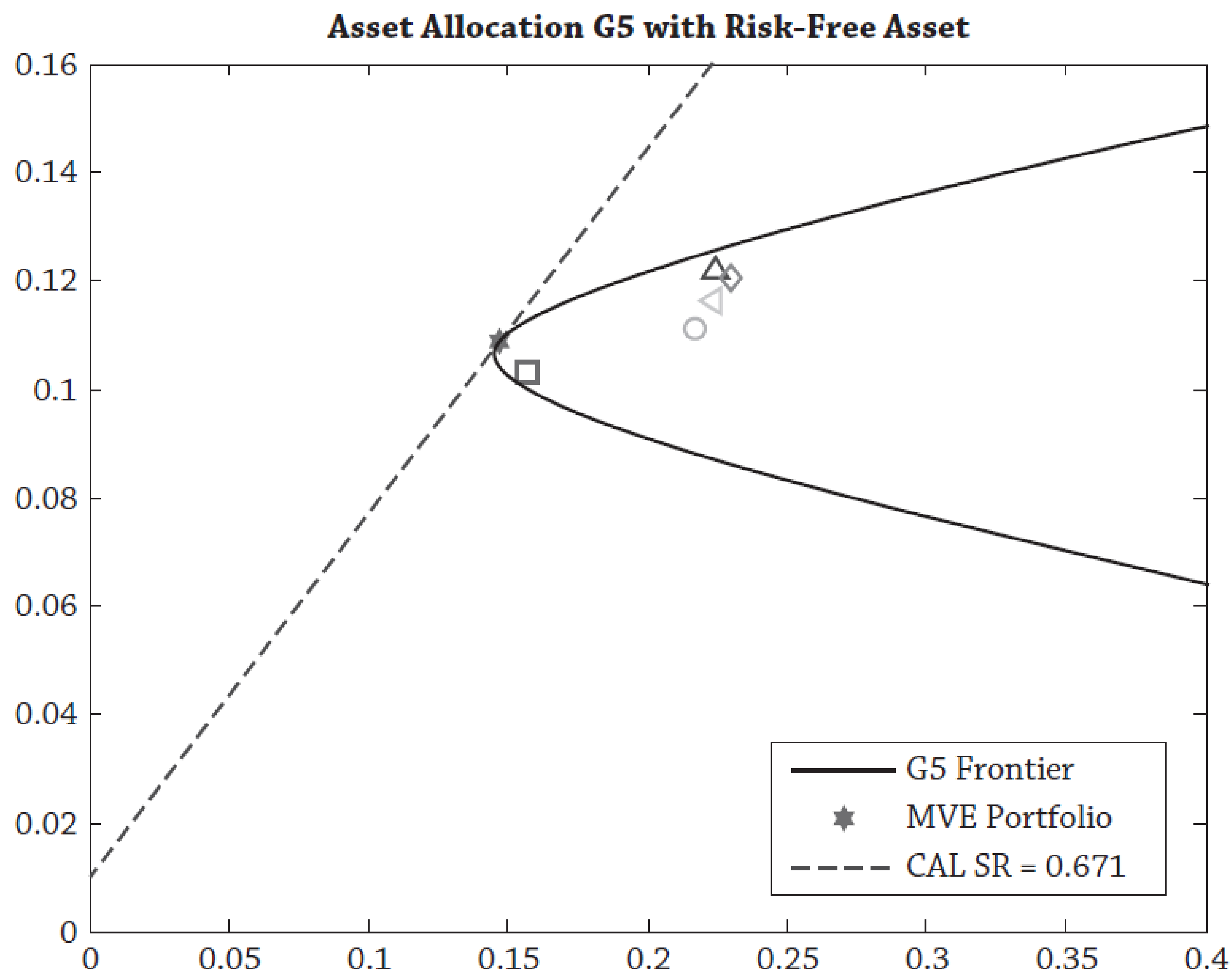


Figure 3.10

Source: Ang (2014)

n risky assets + risk-free asset

Asset Allocation G5 with Risk-Free Asset

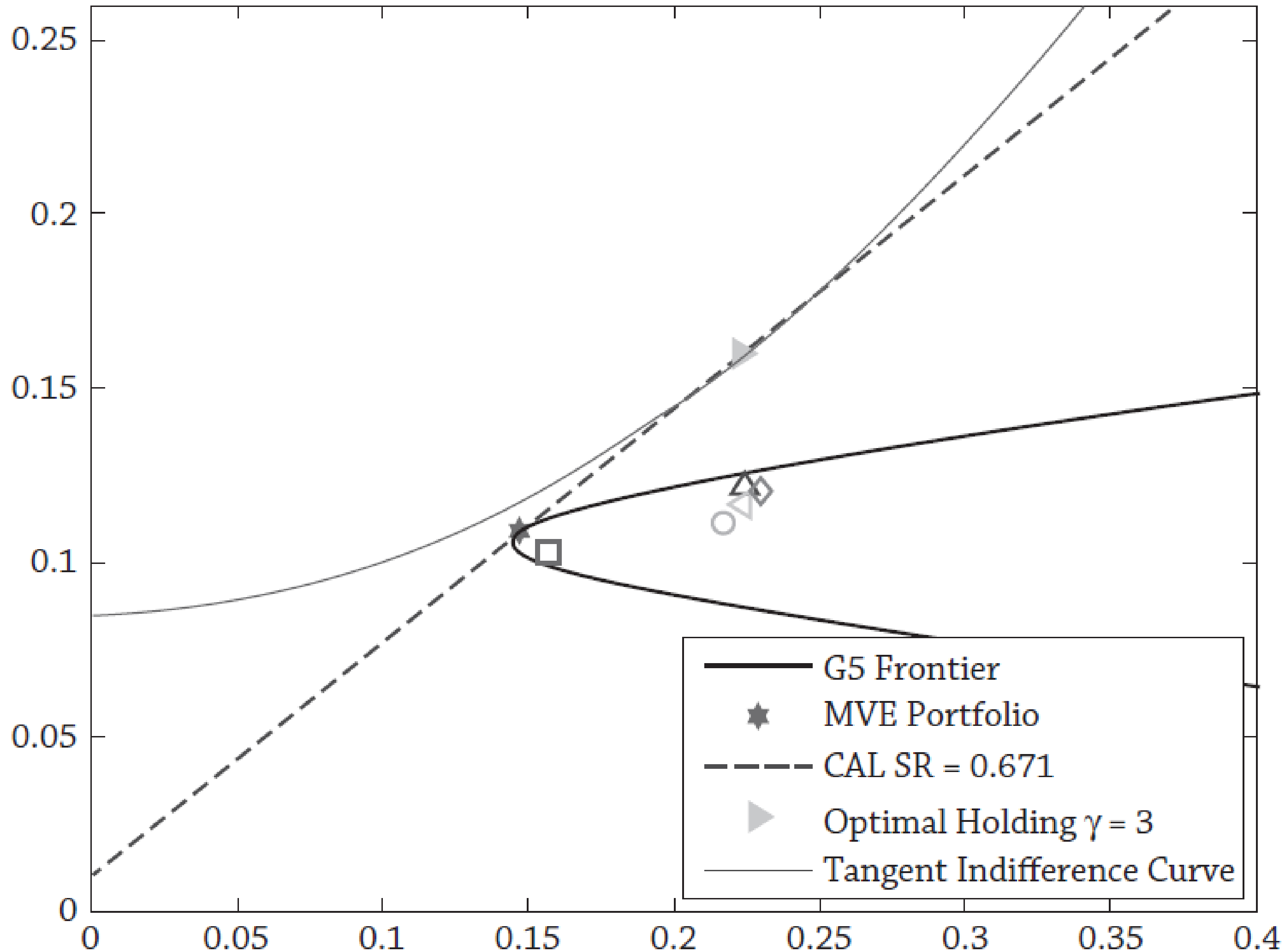


Figure 3.11

Source: Ang (2014)

Global stock return correlation

Table 7.2: Major stock markets summary risk/return statistics 1.1.1996–12.31.2006[a]

	FRA	GER	Japan	UK	US	Brazil	Korea
Average monthly return	0.71%	0.65%	− 0.21%	0.34%	0.57%	1.57%	0.63%
Standard deviation	5.9%	7.2%	5.6%	4.1%	4.5%	10.4%	11.0%
Correlation table							
France	1						
Germany	0.90	1					
Japan	0.45	0.43	1				
UK	0.79	0.79	0.49	1			
US	0.74	0.77	0.37	0.79	1		
Brazil	0.58	0.60	0.27	0.61	0.65	1	
Korea	0.40	0.39	0.45	0.51	0.48	0.39	1

[a]Authors' calculations.

Source: Danthine and Donaldson (2014)

Global stock return correlation

Table 7.3: Major stock markets summary risk/return statistics 1.1.2002–12.31.2006[a]

	FRA	GER	Japan	UK	US	Brazil	Korea
Average monthly return	− 0.40%	− 0.38%	0.25%	− 0.19%	− 0.15%	1.08%	1.36%
Standard deviation	5.9%	7.9%	5.2%	4.1%	4.3%	8.2%	7.4%
Correlation table							
France	1						
Germany	0.95	1					
Japan	0.53	0.49	1				
UK	0.90	0.88	0.51	1			
US	0.89	0.89	0.49	0.87	1		
Brazil	0.67	0.65	0.43	0.67	0.72	1	
Korea	0.74	0.74	0.56	0.71	0.72	0.54	1

[a]Authors' calculations.

Source: Danthine and Donaldson (2014)

Global stock return correlation

Table 7.5: Summary risk/return statistics for major stock markets during the Financial Crisis, 1.1.2008–12.31.2012[a]

		FRA	GER	Japan	UK	US	Italy	Canada
Average monthly return		−0.09%	0.079%	−0.21%	0.05%	0.30%	−0.72%	0.31%
Monthly standard deviation		8.28%	8.87%	5.26%	6.55%	5.47%	9.40%	7.57%
Correlation table								
	FRA	1						
	GER	0.95	1					
	Japan	0.76	0.78	1				
	UK	0.92	0.89	0.79	1			
	US	0.89	0.90	0.77	0.91	1		
	Italy	0.97	0.92	0.75	0.85	0.85	1	
	Canada	0.82	0.81	0.73	0.78	0.86	0.78	1

[a]We thank G. Bekaert for providing these statistics to us.

Source: Danthine and Donaldson (2014)

Practical issues in modern portfolio theory

- MPT produces unrealistic portfolio allocation with extreme long/short positions in stocks
- With short-sale constraint, this problem can be avoided but this will restrict investors from allocating their positions in asset universe
- Also, the previous result assumes that past risks/returns represent those of the future
- This leads to the outperformance of an equal-weighted portfolio over the optimal risky portfolio

Practical issues in modern portfolio theory

Table A.7.1: Portfolio proportions when $\sigma_p = 5\%$ and short sales are not allowed

Short-Sale Constraints?	No	No	No	No	No
Horizon	120	120	120	120	120
France	122.28%	99.24%	89.59%	79.41%	79.79%
Germany	− 54.85%	− 56.23%	− 67.67%	− 71.09%	− 71.43%
Japan		− 31.24%	− 31.25%	− 21.86%	− 21.22%
UK	32.57%	88.23%	15.69%	20.93%	21.05%
US			93.64%	78.10%	79.06%
Brazil				14.51%	15.29%
Korea					− 2.55%
Mean monthly return	0.62%	0.70%	0.85%	0.89%	0.89%
SD of monthly rate	5.00%	5.00%	5.00%	5.00%	5.00%

Source: Danthine and Donaldson (2014)

Practical issues in modern portfolio theory

Table A.7.2: Portfolio proportions when $\sigma_P = 5\%$ and short sales are permitted.

Short-Sale Constraints?	Yes	Yes	Yes	Yes	Yes
Horizon	120	120	120	120	120
France	62.99%	62.99%	59.09%	17.04%	17.04%
Germany	0.00%	0.00%	0.00%	0.00%	0.00%
Japan	0.00%	0.00%	0.00%	0.00%	0.00%
UK	37.01%	37.01%	0.00%	0.00%	0.00%
US			40.91%	68.82%	68.82%
Brazil				14.14%	14.14%
Korea					0.00%
Mean monthly return	0.57%	0.57%	0.65%	0.74%	0.74%
SD of monthly rate	5.00%	5.00%	5.00%	5.00%	5.00%

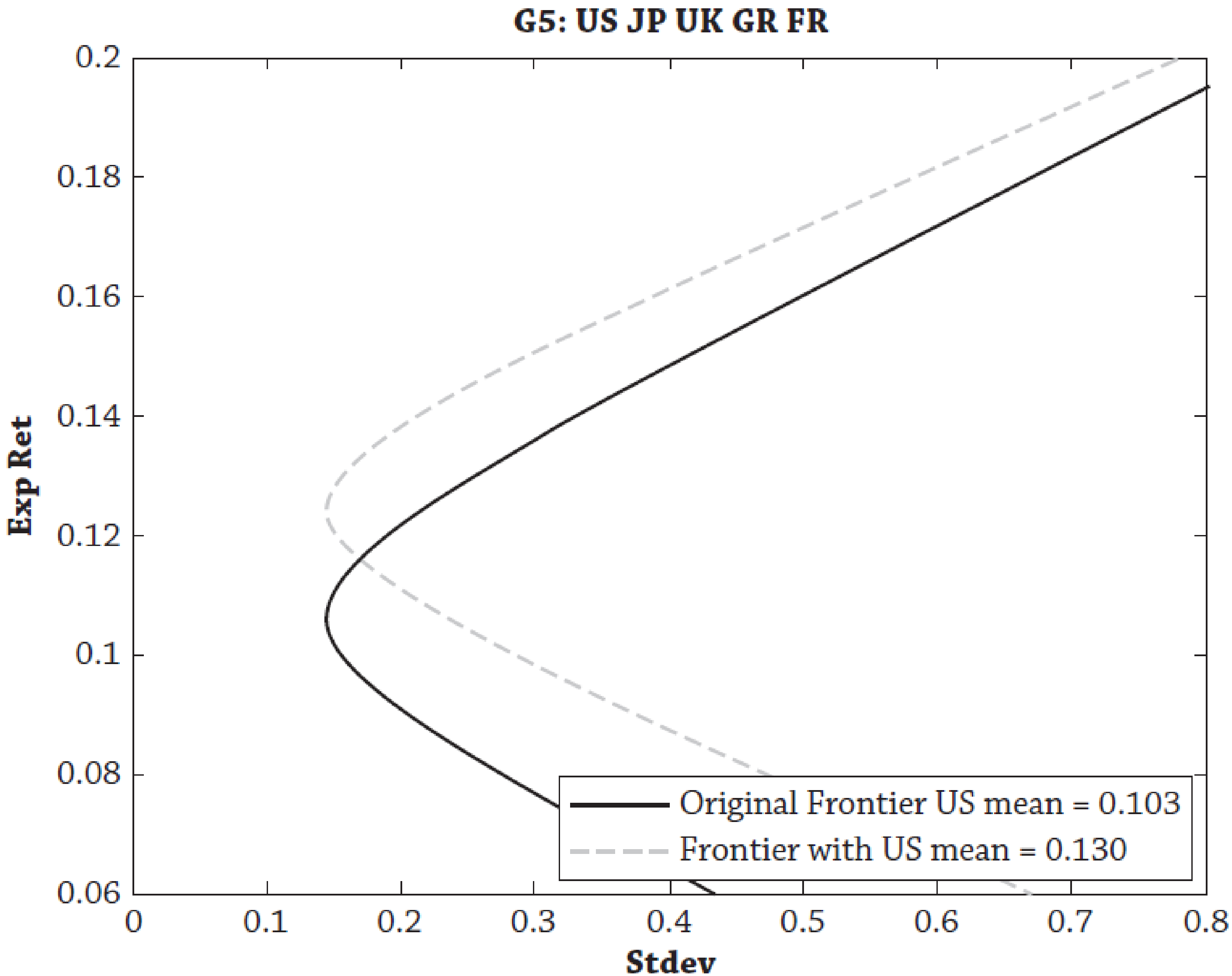
Source: Danthine and Donaldson (2014)

Practical issues in modern portfolio theory

	U.S. mean = 10.3%	U.S. mean = 13.0%
U.S.	−0.0946	0.4101
JP	0.2122	0.3941
U.K.	0.4768	0.0505
GR	0.1800	0.1956
FR	0.2257	−0.0502

Source: Ang (2014)

Practical issues in modern portfolio theory



Source: Ang (2014)

Figure 3.13