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EE 325 Section 3 Take home quiz 2 February 27th, 2020 ☺

Data on X_i and Y_i are given in the table

i	X_i	Y_i
1	4	6
2	3	5
3	2	4
4	8	10
5	1	3

Suppose you decide to fit the following model

$Y_i = \beta_1 + \beta_2 X_i + u_i$ where $u_i \sim NIID(0, \sigma^2)$

$\sum X_i = 18, \sum Y_i = 28$

$\bar{X} = 3.6, \bar{Y} = 5.6$

$\sum (X_i - \bar{X})^2 = 29.2$

$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 29.2$

$\sum X_i Y_i = 130, \sum X_i \sum Y_i = 504, \left(\sum X_i\right)^2 = 94$

Compute estimators of β_1 and β_2 $(\hat{\beta}_1 and \hat{\beta}_2)$.

Interpret the regression.

$\bar{X} = \frac{\sum x_i}{N} = \frac{18}{5} = 3.6$ $\bar{Y} = \frac{\sum y_i}{N} = \frac{28}{5} = 5.6$

$\hat{u}_i$ ($y_i - \hat{y}_i$)	i	X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$ (29.2)
0.4	1	4	6	0.4	0.4	0.16
-0.6	2	3	5	-0.6	-0.6	0.36
-1.6	3	2	4	-1.6	-1.6	2.56
4.4	4	8	10	4.4	4.4	19.36
-2.6	5	1	3	-2.6	-2.6	6.76

$$y_i = \beta_1 + \beta_2 X_i + u_i$$

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$$

$$\hat{y}_i = 5.6 + \cancel{0} + \cancel{\hat{u}_i}^{\,0}$$

$$u_i \sim NIID(0, \sigma)$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 5.6 - 3.6\hat{\beta}_2 = 5.6 - 3.6 = 2$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{29.2}{29.2} = 1$$

$$u_i \sim NIID(0, \sigma^2)$$

$$\hat{y}_i = 5.6, \quad \hat{\beta}_1 = 2, \quad \hat{\beta}_2 = 1$$

∴ The value of the estimator of β_1 & β_2 ($\hat{\beta}_1$ & $\hat{\beta}_2$) are 2 and 1, respectively.

Interpret $\hat{\beta}_2$, from $\hat{\beta}_2 = 1$ this means if X increase by 1 unit, on average, Y will increase 1 unit.

Interpret $\hat{\beta}_1$, suppose X = 0, on average, Y will equal to 2.