$f_x = \underline{f_x(x,y)}$

#concentrate

$$df_x = \underbrace{\frac{\partial f_x(x,y)}{\partial x}}_{f_{xx}} \cdot dx + \underbrace{\frac{\partial f_x(x,y)}{\partial y}}_{f_{xy}} \cdot dy$$

$$\left.\begin{aligned} (df_x)\,dx &= \left[f_{xx}\,dx + f_{xy}\,dy \right] dx \\ (df_y)\,dy &= \left[f_{yx}\,dx + f_{yy}\,dy \right] dy \end{aligned}\right\} d^2f$$

$$d\,(f_x\,dx + f_y\,dy)$$

$$d\,(a+b)$$

$$da + db$$

$$d(f_x\,dx) + d(f_y\,dy)$$

$$(df_x)\,dx + (df_y)\,dy$$

$$d^2f = f_{xx}(dx)^2 + f_{xy}\cdot dy \cdot dx + f_{yx}\, dx \cdot dy + f_{yy}(dy)^2$$

$$f_{xy} = f_{yx}$$

$$= \left. \begin{aligned} &f_{xx}(dx)^2 + 2 f_{xy}\cdot dy\, dx \\ &+ f_{yy}(dy)^2 \end{aligned} \right\} \begin{aligned} &> 0 \\ &< 0 \end{aligned}$$

$\cancel{(dx,)}$ (dx, dy)

$$d^2f = \begin{bmatrix} dx & dy \end{bmatrix} \underline{\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}} \begin{bmatrix} dx \\ dy \end{bmatrix}$$

2^{nd} order Derivative matrix

"Hessian Matrix"

H: Positive def → f' is ~~local m~~ convex

H: Negative def → " —————— " Concave.

II Suppose $f(x,y) = \frac{x^3}{3} + y^2 - 2x + 2y - 2xy$

Locate optimizer(s).

Step 1: $\nabla f = 0$

$$\left.\begin{aligned} f_x &= x^2 - 2 - 2y = 0 \quad \text{— (1)} \\ f_y &= 2y + 2 - 2x = 0 \quad \text{— (2)} \end{aligned}\right\} \quad \frac{(x^*, y^*)}{(1) + (2)}$$

(1) + (2) $\quad x^2 - 2x = 0$

$x(x-2) = 0 \rightarrow x = 0, 2$

$y = -1; 1$

(1) $(0, -1)$?

(2) $(2, 1)$ min

step 2 : d^2f

$$\exists\ H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{bmatrix} 2x & -2 \\ -2 & 2 \end{bmatrix}$$

$$(x^*, y^*) = (2,1) \Rightarrow H = \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix}$$

$|H_1| = 4 > 0$

$|H_2| = 4(2) - (-2)(-2) = 8 - 4 = 4 > 0$ Local min

$F(x=2, y=1)$ is convex at that point / ~~stain~~ stationary

$$H = \begin{bmatrix} 0 & -2 \\ -2 & 2 \end{bmatrix}$$ negativ

$H_1 = 0 < 0$

$H_2 = -4 > 0$

$(0, -1)$ have no idea

Question?

$$f(x,y) = x^2 - 2xy + 3y^2 + 2x - 2y$$

$$x = \frac{\begin{vmatrix} -2 & -2 \\ 2 & 6 \end{vmatrix}}{8}$$

<u>step 1</u> F.O.C. $\longrightarrow$ stationary point of f^2

another $\rightarrow$ $\begin{pmatrix} 2 & -2 \\ -2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$$\nabla f(x,y) = 0$$

$$f_x = 2x - 2y + 2 = 0 \quad \text{— (1)}$$

$$f_y = -2x + 6y - 2 = 0 \quad \text{— (2)}$$

one way

(x^*, y^*) solve for (1) + (2), simultaneously.

max/m

(1) + (2) $\quad 4y = 0 \Rightarrow y^* = 0$

$\Rightarrow x^* = -1$

$(x^*, y^*) = (-1, 0)$

$(x^*, y^*) = (-1; 0)$ stationary point.

↳ global minimize too.

H is always positive definite $\forall (x,$

step 2 - $d^2 f$

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 6 \end{bmatrix}$$

Positive Def $\to d^2 f > 0$

$$|H_1| = |2| > 0$$

→ f: convex at $(x^*; y^*) \to$ min point

$$|H_2| = \begin{vmatrix} 2 & -2 \\ -2 & 6 \end{vmatrix} = 2 \cdot 6 - (-2)(-2) = 12 - 4 = 8 > 0$$

$\boxed{\text{Local optimizer}}$

Step 1 $(x^*, y^*) \rightarrow \nabla F = 0$

$$\left.\begin{array}{l} \nabla F = 0 \\ \dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial y} = 0 \end{array}\right\} (x^*, y^*) \Downarrow \text{stationary point}$$

<u>Step 2</u>: f: Convex/Concave (x^*, y^*)

$d^2f > 0$ (convex) $\rightarrow |H_1| > 0$; $|H_2| > 0$

< 0 (concave) $\rightarrow |H_1| < 0$; $|H_2| > 0$

Pattern doesn't match $\rightarrow$ ~~no~~ inconclusive

<u>Global</u>: local $\rightarrow$ global ; globally Concave $\forall (x$ negativ

globally convex $\forall (x$ positive