

1). Firm 1: $\pi_1 = TR_1 - TC_1$
 $= (a - bQ_1 - bQ_2 - bQ_3)Q_1 - C_1$
 FOC: $\frac{\partial \pi_1}{\partial Q_1} \Rightarrow a - 2bQ_1 - bQ_2 - bQ_3 = 0$
 $a - bQ_2 - bQ_3 = 2bQ_1$
 $Q_1 = \frac{a - bQ_2 - bQ_3}{2b}$

Substitute Q_1
 $Q_1 = \frac{a - b\left(\frac{a+bQ_3}{3b}\right) - bQ_3}{2b} = \frac{3a - a + bQ_3 - 3bQ_3}{6b}$
 $Q_1 = \frac{2a - 2bQ_3}{6b} \Rightarrow \frac{a - bQ_3}{3b} \quad \text{--- ①}$

Firm 2: $\pi_2 = TR_2 - TC_2$
 $= (a - bQ_1 - bQ_2 - bQ_3)Q_2 - C_2$
 FOC: $\frac{\partial \pi_2}{\partial Q_2} \Rightarrow a - bQ_1 - 2bQ_2 - bQ_3 = 0$
 $a - bQ_1 - bQ_3 = 2bQ_2$

Substitute Q_1
 $2bQ_2 = a - b\left(\frac{a - bQ_2 - bQ_3}{2b}\right) - bQ_3$
 $= \frac{2a - a + bQ_2 + bQ_3 - 2bQ_3}{2}$
 $2bQ_2 = \frac{a + bQ_2 - bQ_3}{2}$
 $4bQ_2 = a + bQ_2 - bQ_3$
 $Q_2 = \frac{a + bQ_3}{3b} \quad \text{--- ②}$

Equilibrium price $\Rightarrow P = a - bQ$
 $= a - b\left(\frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b}\right)$
 $= a - \frac{3a}{4}$
 $= \frac{1}{4}a \text{ or } 0.25a \quad \#$

Firm Profit: $\pi_1 = P(Q_1) - C_1$
 $= 0.25a \cdot \frac{a}{4b} - C_1$
 $= \frac{a^2}{16b} - C_1$

$\pi_2 = P(Q_2) - C_2$
 $= 0.25a - \frac{a}{4b} - C_2$
 $= \frac{a^2}{16b} - C_2$

$\pi_3 = P(Q_3) - C_3$
 $= 0.25a - \frac{a}{4b} - C_3$
 $= \frac{a^2}{16b} - C_3 \quad \#$

Firm 3: $\pi_3 = TR_3 - TC_3$
 $= (a - bQ_1 - bQ_2 - bQ_3)Q_3 - C_3$
 FOC: $\frac{\partial \pi_3}{\partial Q_3} \Rightarrow a - bQ_1 - bQ_2 - 2bQ_3 = 0$
 $a - bQ_1 - bQ_2 = 2bQ_3$

Substitute Q_1 & Q_2
 $Q_3 = \frac{a - b\left(\frac{a+bQ_3}{3b}\right) - b\left(\frac{a+bQ_3}{3b}\right)}{2b}$
 $= \frac{3a - a - bQ_3 - a - bQ_3}{6b}$
 $= \frac{a + 2bQ_3}{6b}$
 $6bQ_3 = a + 2bQ_3$
 $4bQ_3 = a$
 $Q_3 = \frac{a}{4b} \quad \text{--- ③}$

2. Assume that $Q_1 + Q_2 + Q_3 + \dots + Q_n = A$

$$P = a - b(Q_1 + Q_2 + Q_3 + \dots + Q_n)$$

$$= a - bQ_1 - bQ_2 - bQ_3 - \dots - bQ_n$$

$$\pi_1 = (a - bQ_1 - bQ_2 - bQ_3 - \dots - bQ_n) \cdot Q_1 - C_1$$

$$\pi_n = (a - bQ_1 - bQ_2 - bQ_3 - \dots - bQ_n) \cdot Q_n - C_n$$

$$\frac{\partial \pi_1}{\partial Q_1} = a - 2bQ_1 - bQ_2 - bQ_3 - \dots - bQ_n = 0$$

$$Q_1 = \frac{a - b(Q_2 + Q_3 + \dots + Q_n)}{2b}$$

$$Q_n = \frac{a}{2b} - b(Q_1 + Q_2 + \dots + Q_{n-1})$$

3. If $n \rightarrow \infty$; it will make Q_i nearly zero

; it will make $\pi_i = -C_i \Rightarrow$ lost the profit

If $n=1$; it will make $Q_i = a/2b$

; it will make $\pi_i = \frac{a^2}{4b} - C_i$