Solution to Question Book for Lecture Note 7

Introduction to Risk, Return and Portfolio Theory

Problem 1: Explain or qualify the following problems as necessary:

1.1 *Diversifiable Risk.* In light of what you've learned about market versus diversifiable (unique) risks, explain why an insurance company has no problem in selling life insurance to individuals but is reluctant to issue policies insuring against flood damage to residents of coastal areas. Why don't the insurance companies simply charge coastal residents a premium that reflects the actuarial probability of damage from hurricanes and other storms?

1.2 *Unique vs. Market Risk.* Figure 4.14 plots monthly rates of return from 1993 to 1999 for the Snake Oil mutual fund. Was this fund well-diversified? Explain.

1.3 *Risk and Return.* Suppose that the risk premium on stocks and other securities did in fact rise with total risk (that is, the variability of returns) rather than just market risk. Explain how investors could exploit the situation to create portfolios with high expected rates of return but low levels of risk.

Problem 2: A stock's return has the following distribution:

Demand for the Company's Products	**Probability of This Demand Occurring**	**Rate of Return If This Demand Occurs (%)**
Weak	0.1	−50%
Below average	0.2	-5
Average	0.4	16
Above average	0.2	25
Strong	0.1	60

Calculate the stock's expected return, standard deviation, and coefficient of variation.

Sol:

Expected return	**Variance**	**Standard Deviation**	**Coefficient of Variation**
11.4	712.44	26.69	2.3414

Problem 3: The market and Stock J have the following probability distributions:

Probability	R_M	R_J
0.3	15%	20%
0.4	9	5
0.3	18	12

a. Calculate the expected rates of return for the market and Stock J.
b. Calculate the standard deviations for the market and Stock J.
c. Calculate the coefficients of variation for the market and Stock J.

Sol:

a. Expected rates of return for the market = 13.5%
   Expected rates of return for Stock J = 11.6%
b. Standard deviations for the market = 14.85
   Standard deviations for Stock J = 38.64
c. Coefficients of variation for the market = 0.28545
   Coefficients of variation for the market = 0.535871

Problem 4: Consider the following two scenarios for the economy, and the returns in each scenario for the market portfolio, an aggressive stock A, and a defensive stock D.

	Rate of Return		
Scenario	**Market Aggressive**	**Stock A**	**Defensive Stock D**
Bust	–8%	–10%	–6%
Boom	32	38	24

a. If each scenario is equally likely, find the expected rate of return on the market portfolio and on each stock.
b. Which stock seems to be a better buy based on your answers to (a)?

Sol:

a. Expected return on the market portfolio = 12%
   Expected return on Stock A = 14%
   Expected return on Stock D = 9%
b. Standard Deviation of Stock A = 24
   Standard Deviation of Stock D = 15
   Coefficient of variation of Stock A = 1.7143
   Coefficient of variation of Stock D = 1.6667
   Therefore, Stock D seems to a better buy stock.

Problem 5: Consider the possible rates of return that you might earn next year on a $50,000 investment in stock A or on a $50,000 investment in stock B, depending upon the states of the economy: recession, normal, and prosperity.

For stock A:

State of Economy	Return (R_i)	Probability (P_i)
Recession	-5%	0.2
Normal	20%	0.6
Prosperity	40%	0.2

For stock B:

State of Economy	Return (R_i)	Probability (P_i)
Recession	10%	0.2
Normal	15%	0.6
Prosperity	20%	0.2

Calculate the stocks' expected return, variance, and standard deviation. Then, identify which stock is more attractive to invest?

Sol:

Then the expected rate of return ($\bar{r}$) for stock A is computed as follows:

$$\bar{r} = \sum_{i=1}^{n} r_i p_i = (-5\%)(0.2) + (20\%)(0.6) + (40\%)(0.2) = 19\%$$

Stock B's expected rate of return is:

$$\bar{r} = (10\%)(0.2) + (15\%)(0.6) + 20\%(0.2) = 15\%$$

		Step 1	Step 2		Step 3
Return (r_i) (%)	Probability (p_i)	$r_i p$ (%)	$(r_i - \bar{r})(\%)$	$(r_i - \bar{r})^2$	$(r_i - \bar{r})^2 p_i(\%)$
−5	0.2	−1	−24	576	115.2
20	0.6	12	1	1	0.6
40	0.2	8	21	441	88.2
		$\bar{r} = 19$			$\sigma^2 = 204$

For stock B:

		Step 1	Step 2		Step 3
Return (r_i) (%)	Probability (p_i)	$r_i p$ (%)	$(r_i - \bar{r})(\%)$	$(r_i - \bar{r})^2$	$(r_i - \bar{r})^2 p_i(\%)$
10	0.2	2	−5	25	5
15	0.6	9	0	0	0
20	0.2	4	5	25	5
		$\bar{r} = 15$			$\sigma^2 = 10$

Problem 6: Assuming the following probability distribution of the possible returns, calculate the expected return and the standard deviation of the returns.

Probability (P_i)	*Return (R_i)*
0.1	-20%
0.2	5%
0.3	10%
0.4	25%

Sol:

$$r = \sum r_i p_i$$

$$\sigma = \sqrt{\sum (r_i - \bar{r})^2 p_i}$$

It is convenient to set up the following table:

r_i (%)	p_i	$r_i p_i$ (%)	$(r_i - \bar{r})(\%)$	$(r_i - \bar{r})^2$	$(r_i - \bar{r})^2 p_i$ (%)
−20	0.1	−2	−32	1,024	102.4
5	0.2	1	−7	49	9.8
10	0.3	3	−2	4	1.2
25	0.4	10	13	169	67.6
		$\bar{r} = 12$			$\sigma^2 = 181$

Since $\sigma^2 = 181$, $\sigma = \sqrt{181} = 13.45\%$.

Problem 7: Stocks A and B have the following probability distributions of possible future returns:

Probability (pi)	*A (%)*	*B (%)*
0.1	-15	-20
0.2	0	10
0.4	5	20
0.2	10	30
0.1	25	50

(*a*) Calculate the expected rate of return for each stock and the standard deviation of returns for each stock, (*b*) Calculate the coefficient of variation, (*c*) Which stock is less risky? Explain.

Sol:

(*a*) For stock A:

r_i (%)	p_i	r_ip_i (%)	$(r_i - \bar{r})$(%)	$(r_i - \bar{r})^2$	$(r_i - \bar{r})^2 p_i$ (%)
−15	0.1	−1.5	−20	400	40
0	0.2	0	−5	25	5
5	0.4	2	0	0	0
10	0.2	2	5	25	5
25	0.1	2.5	20	400	40
		$\bar{r} = 5.0$			$\sigma^2 = 90$

Since $\sigma^2 = 90$, $\sigma = \sqrt{90} = 9.5\%$.

For stock B:

r_i (%)	p_i	r_ip_i (%)	$(r_i - \bar{r})$(%)	$(r_i - \bar{r})^2$	$(r_i - \bar{r})^2 p_i$ (%)
−20	0.1	−2	−39	1,521	152.1
10	0.2	2	−9	81	16.2
20	0.4	8	1	1	0.4
30	0.2	6	11	121	24.2
50	0.1	5	31	961	96.1
		$\bar{r} = 19$			$\sigma^2 = 289$

Since $\sigma^2 = 289$, $\sigma = \sqrt{289} = 17\%$.

(*b*) The coefficient of variation is $\sigma/\bar{r}$. Thus, for stock A:

$$\frac{9.5\%}{5\%} = 1.9$$

For stock B:

$$\frac{17.0\%}{19\%} = 0.89$$

(*c*) Stock B is less risky than stock A since the coefficient of variation (a measure of relative risk) is smaller for stock B.

Problem 8: Ken Parker must decide which of two securities is best for him. By using probability estimates, he computed the following statistics:

Statistic	Security X	Security Y
Expected return	12%	8%
Standard deviation	20%	10%

(*a*) Compute the coefficient of variation for each security, and (*b*) explain why the standard deviation and coefficient of variation give different rankings of risk. Which method is superior and why?

Sol:

(a) For the X coefficient of variation $(\sigma / \bar{r})$ is 20/12=1.67. For Y it is 10/8=1.25.

(b) Unlike the standard deviation, the coefficient of variation considers the standard deviation of securities relative to their average return. The coefficient of variation is therefore the more useful measure of relative risk. The lower the coefficient of variation, the less risky the security relative to the expected return. Thus, in this problem, security Y is relatively less risky than security X.

Problem 9: A portfolio consists of assets A and B. Asset A makes up one-third of the portfolio and has an expected return of 18 percent. Asset B makes up the other two-thirds of the portfolio and is expected to earn 9 percent. What are the expected return and risk on this portfolio?

Sol:

Portfolio Expected Return = (1/3)x18% + (2/3)x9% = 12%

Problem 10: The securities of firms A and B have the expected return and standard deviations given below; the expected correlation between the two stocks (ρ_{AB}) is 0.1.

Stock	*Expected Return*	*Standard Deviation*
A	14%	20%
B	9%	30%

Compute the return and risk for each of the following portfolios: (*a*) 100 percent A; (*b*) 100 percent B; (*c*) 60 percent A– 40 percent B; and (*d*) 50 percent A–50 percent B.

Additional Question: assume the expected correlation between the two stocks (ρ_{AB}) = -1.0. How does this have an effect on previous answers?

Sol:

(*a*) 100 percent A} $\bar{r} = 14\%;\ \sigma = 20\%;\ \sigma/\bar{r} = \frac{20}{14} = 1.43$

(*b*) 100 percent B: $\bar{r} = 9\%;\ \sigma = 30\%;\ \sigma/\bar{r} = \frac{30}{9} = 3.33$

(*c*) 60 percent A – 40 percent B:

$$r_p = w_A r_A + w_B r_B = (0.6)(14\%) + (0.4)(9\%) = 12\%$$

$$\sigma_p = \sqrt{w_A^2\sigma_B^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho_{AB}\sigma_A\sigma_B}$$

$$= \sqrt{(0.6)^2(0.2)^2 + (0.4)^2(0.3)^2 + 2(0.6)(0.4)\rho_{AB}(0.2)(0.3)}$$

$$= \sqrt{0.0144 + 0.0144 + 0.0288\rho_{AB}} = \sqrt{0.0288 + 0.0288(0.1)} = \sqrt{0.03168} = 0.1780 = 17.8\%$$

(d) 50 percent A – 50 percent B:

$$r_p = (0.5)(14\%) + (0.5)(9\%) = 11.5\%$$

$$\sigma_p = \sqrt{(0.5)^2(0.2)^2 + (0.5)^2(0.3)^2 + 2(0.5)(0.5)\rho_{AB}(0.2)(0.3)}$$
$$= \sqrt{0.01 + 0.0225 + 0.03\rho_{AB}} = \sqrt{0.0325 + 0.03\rho_{AB}}$$
$$= \sqrt{0.0325 + 0.03(0.1)} = \sqrt{0.0355} = 0.1884 = 18.84\%$$

Problem 11: What is the standard deviation of the following two-stock portfolio?

	Weighting	Standard Deviation	Correlation
Stock A	60%	0.11	0.7
Stock B	40%	0.14	

Sol·

portfolio risk is:

$$\sigma_p = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2\rho_{AB}w_A w_B \sigma_A \sigma_B}$$
$$= \left[(0.6)^2(0.11)^2 + (0.4)^2(0.14)^2 + 2(0.7)(0.6)(0.4)(0.11)(0.14)\right]^{1/2} = (126.664)^{1/2} = 0.1125 = 11.25\%$$

Problem 12: Consider the following information:

State of Economy	**Probability of State of Economy**	**Rate of Return if State Occurs** Stock A	Stock B	Stock C
Boom	.20	.30	.45	.33
Good	.40	.12	.10	.15
Poor	.30	.01	-.15	-.05
Bust	.10	-.06	-.30	-.09

a. Your portfolio is originally invested 40 percent in A and 60 percent in C. Calculate the expected return, variance, and standard deviation of the portfolio?

b. If you are thinking about adding stock B into your portfolio with 50 percent fraction, how will this affect the portfolio's expected return, variance, and standard deviation?

(Assuming coefficient of correlation of all cases is equal to 1)

Sol:

Expected Return			**Variance**		
Stock A	**Stock B**	**Stock C**	**Stock A**	**Stock B**	**Stock C**
0.105	0.055	0.102	0.013125	0.039909	0.009098

a. Portfolio expected return = 10.32%, Variance = 0.010621, Standard Deviation = 0.103057

b. Portfolio expected return = 7.91%, Variance = 0.022927, Standard Deviation = 0.151415