Topic 3 Part 3

The Theory of Demand (Chapter 5)

Summary

- When the price of a good changes, its quantity demanded changes.

- The total effect on demand (TE) is due to **the sum of two effects**:
  - Income Effect (IE)
  - Substitution Effect (SE).

- That is, **TE = SE + IE.**

Summary

When the price of a good falls…

- Substitution Effect implies that the consumer will ALWAYS buy more of the good.
- Income Effect implies that the real income increases AND
  - the consumer will buy more of the good if it is a normal good.
  - the consumer will buy less of the good if it is an inferior good.

Summary

- To represent these two effects in the consumer choice diagram, **economists adjust the BL with two steps**:

1. **“PIVOT”** to represent the SE
2. **“SHIFT”** to represent the IE

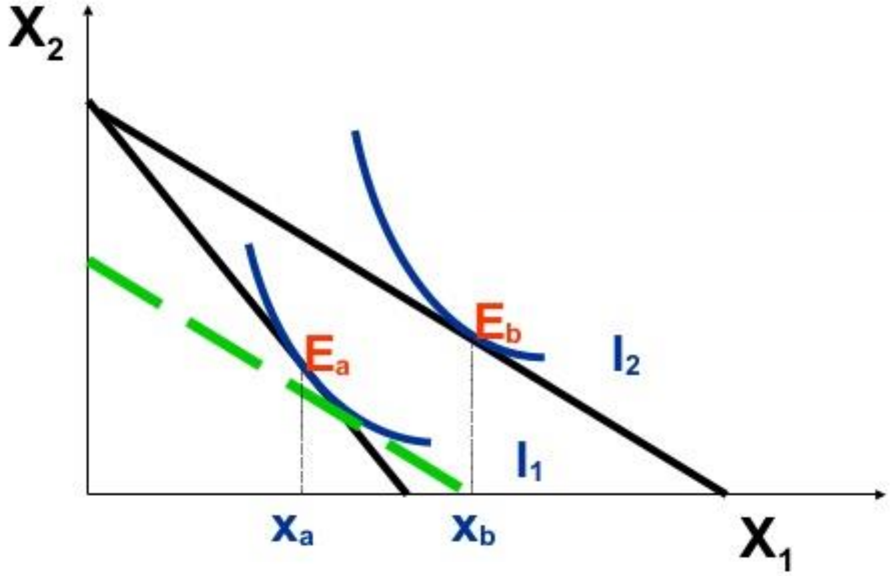
- Moreover, there are two approaches when we try to identify the SE by “pivoting” the BL.
- These approaches are
  - Hicksian Approach
  - Slutskian Approach

Summary

Hicksian Approach

- We find the SE by pivoting the BL (to reflect the price change) along the **original indifference curve**.
- We are essentially asking ourselves:
  Which Bundle after the price change, would keep the consumer at the utility level that he had prior the price change?
- Hicksian Definition of the SE:
  "the change in the demand for a good as its price changes, holding the utility constant".

THE HICKSIAN METHOD

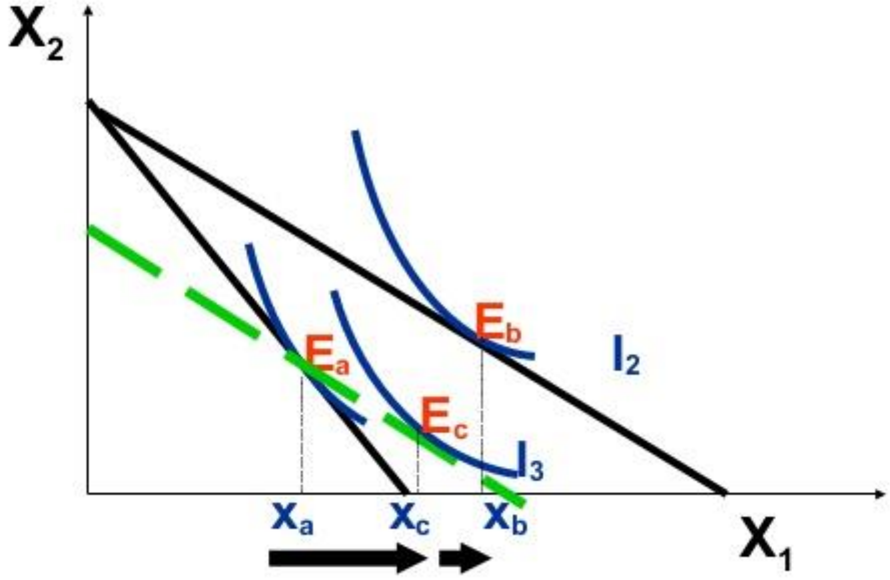


Summary

Slutskian Approach

- We find the SE by pivoting the BL (to reflect the price change) around the **original bundle**.
- We are essentially asking ourselves:
  Which Bundle after the price change, would keep the consumer at the purchasing power that he had prior the price change?
- Slutskian Definition of the SE:
  "the change in the demand for a good as its price changes, holding the purchasing power constant".

THE SLUTSKY METHOD



Slutskian Method Example

Suppose that we have the **demand function for X**:

$$X(Px, I) = 10 + \frac{I}{10Px}.$$

$X(Px, I)$ is the demand of X as a function of Px and I.

Note that

1. **X is a normal good because dX/dI > 0.**
2. **X is an ordinary good because dX/dPx < 0.**

Slutskian Method Example

$$X(Px, I) = 10 + \frac{I}{10Px}.$$

<u>Suppose I = 120 and Px = 3.</u>

We know that **X(3, 120)** = 10 + 120/30 = 14.

(X = 14 maximizes U when I = 120 and Px = 3)

<u>Now, what if Px falls from 3 to 2 per unit?</u>

We know that **X(2, 120)** = 10 + 120/20 = 16.

(X = 16 maximizes U when I = 120 and Px = 2)

Slutskian Method Example

The demand for X increases by 2 units.

That is, we buy more because X is cheaper.

This is the Total Effect (TE) of the price change.

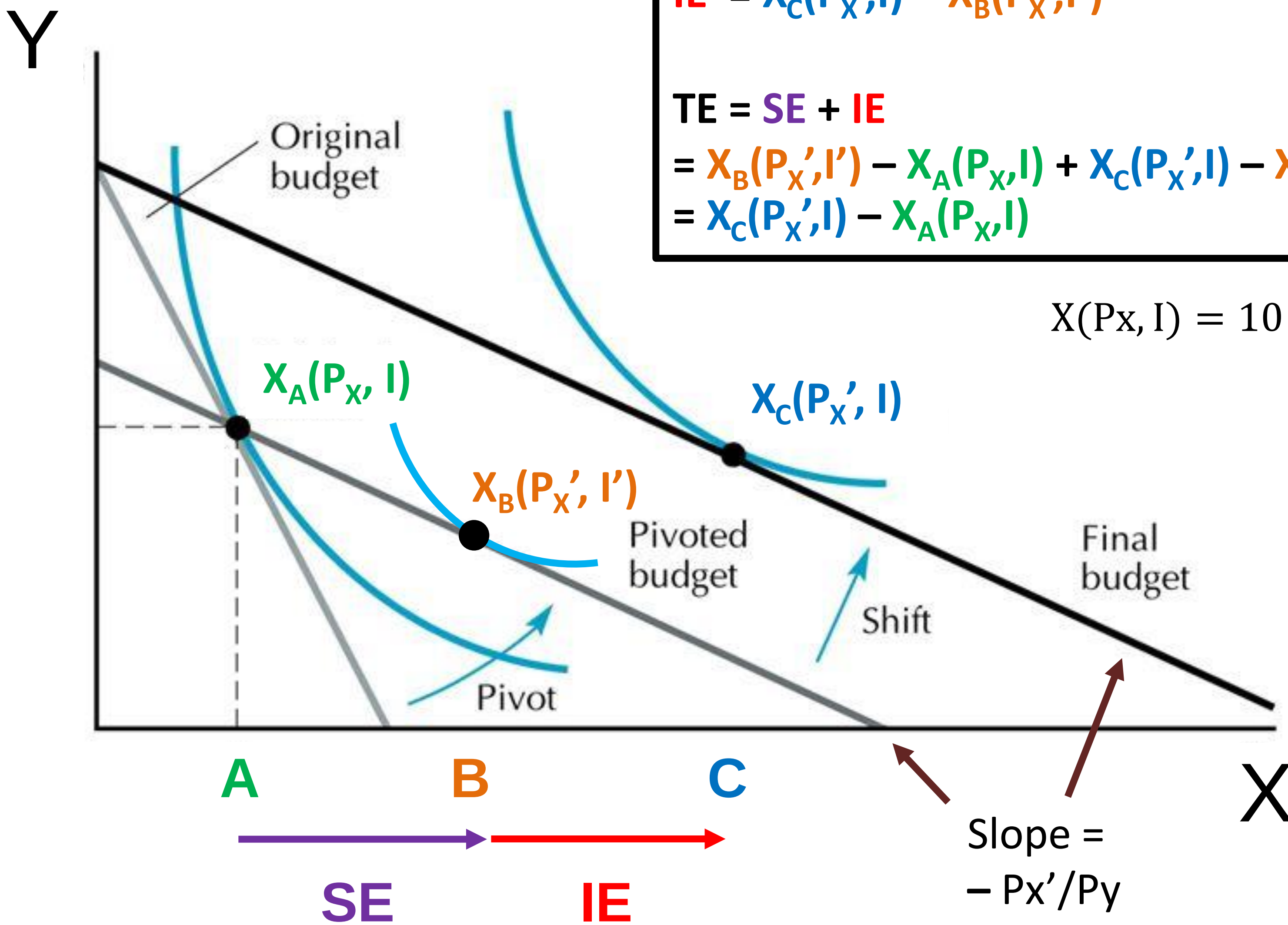
Now, we want to know how much the Income Effect (IE) and the Substitution Effect (SE) are.

Note that TE = SE + IE.

$$SE = X_B(P_X',I') - X_A(P_X,I)$$
$$IE = X_C(P_X',I) - X_B(P_X',I')$$

$$TE = SE + IE$$
$$= X_B(P_X',I') - X_A(P_X,I) + X_C(P_X',I) - X_B(P_X',I')$$
$$= X_C(P_X',I) - X_A(P_X,I)$$

$$X(Px, I) = 10 + \frac{I}{10Px}$$



Pivot and shift

SE = $X_B(2, I') - 14$

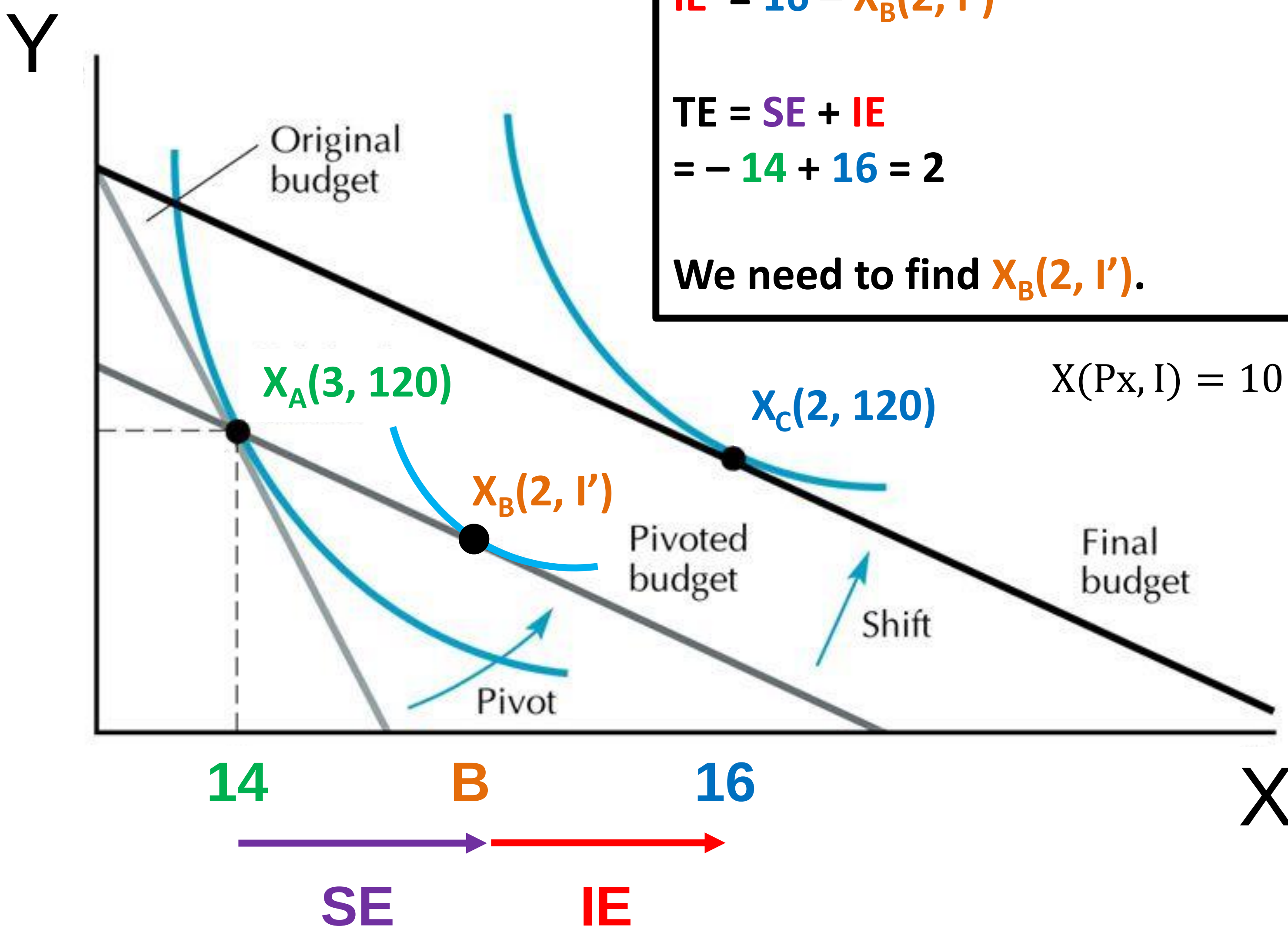
IE = $16 - X_B(2, I')$

TE = SE + IE

$= -14 + 16 = 2$

We need to find $X_B(2, I')$.

$$X(Px, I) = 10 + \frac{I}{10Px}$$



Pivot and shift

Slutskian Method Example

Substitution Effect (holding purchasing power constant)

To find the SE,

1. Find the new income (I') of the pivoted BL.
2. Find the optimal bundle (Bundle B) on this pivoted BL by using the demand equation.

Slutskian Method Example

To find the SE,

1. **Find the new income (I’) of the pivoted BL.**

First, notice that the Pivoted BL is parallel to the Final BL.

<u>This implies that I’ must be less than $120.</u>

Also notice that Bundle A is on the Pivoted BL (where the price has already changed).

<u>This is to keep the purchasing power constant.</u>

Slutskian Method Example

To find the SE,

1. Find the new income (I') of the pivoted BL.

Since Bundle A (X_A, Y_A) is on both BL's, this means

Original BL: $PxX_A + PyY_A = I$ $3(14) + PyY_A = 120$

Pivoted BL: $Px'X_A + PyY_A = I'$ $2(14) + PyY_A = I'$

Both BL's intersect (i.e. they are equal) at Bundle A, solving by equating the two BL's gives I' = 120 – 14 = 106.

Slutskian Method Example

To find the SE,

2. Find the optimal bundle (Bundle B) on this pivoted BL by using the demand equation.

We now use the demand equation to find Bundle B.

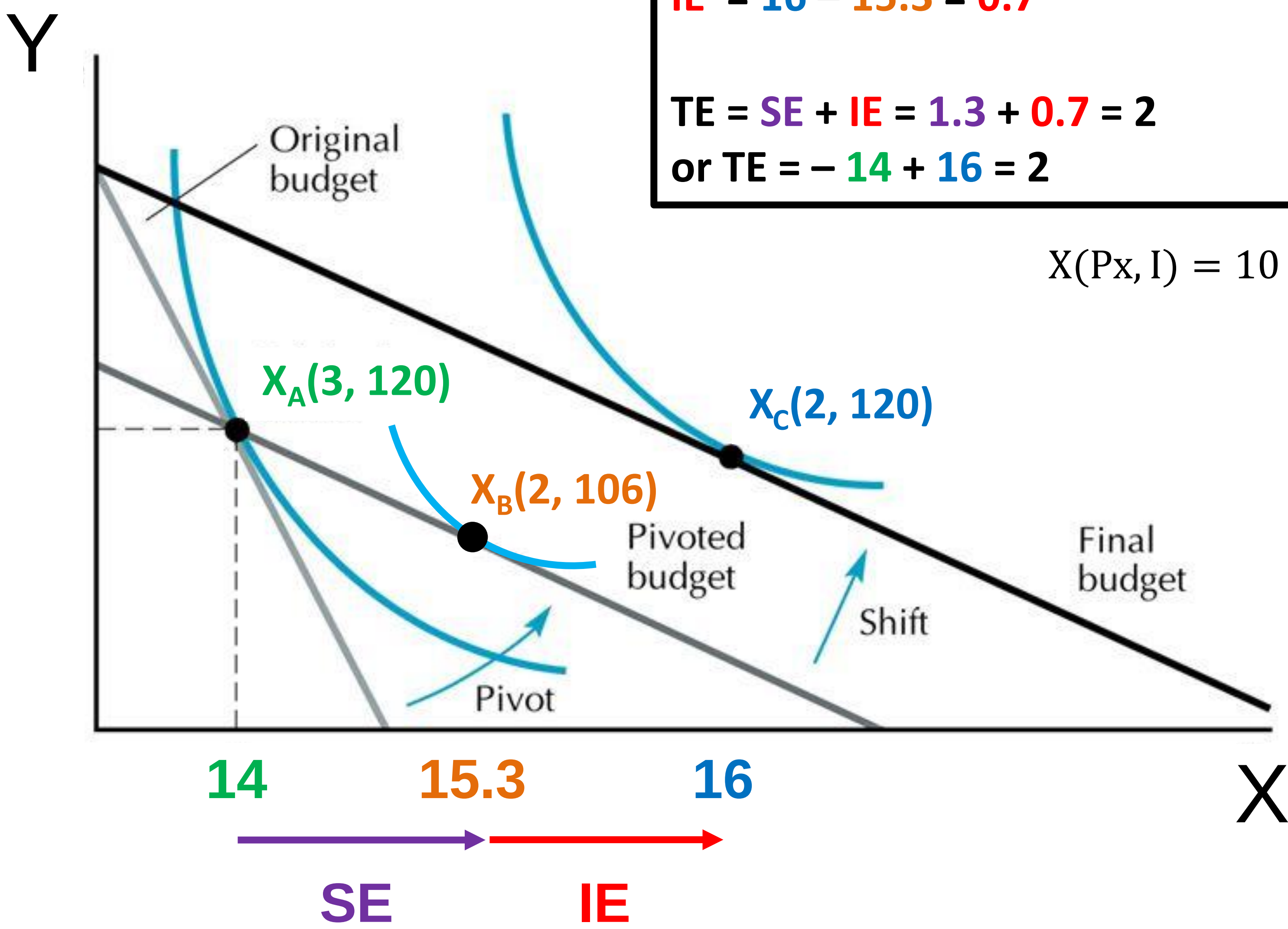
$$X(Px, I) = 10 + \frac{I}{10Px}$$

$$X_B(2, 106) = 10 + \frac{106}{10(2)} = 15.3$$

SE = 15.3 – 14 = 1.3
IE = 16 – 15.3 = 0.7

TE = SE + IE = 1.3 + 0.7 = 2
or TE = – 14 + 16 = 2

$$X(Px, I) = 10 + \frac{I}{10Px}$$



Pivot and shift

Income Compensation for the SE

- In the diagram before, Bundle B is on the BL with a lower income of $106 although the consumer has the income of $120. WHY?

- As previously mentioned in Topic 3 Part 2 Page 23, Slutskian and Hicksian Methods **“compensate”** <u>income</u> of the consumer when identifying the substitution effect.

Income Compensation for the SE

- **"Compensate"** refers to the approach that
  - Slutskian Method gives the consumer just enough money to get back to his original bundle A, so that his purchasing power is constant.
  - Hicksian Method gives the consumer just enough money to get back to his original utility, so that his utility is constant.

Income Compensation for the SE

- However, when Px falls, it is obvious that the consumer can still get back to his original Bundle A, and he should have excess money to buy something else. Thus, his purchasing power is greater after Px falls.
- Since Slutskian Method holds the purchasing power constant, money has to be taken from the consumer to reduce his purchasing power.
- This is why at Pivoted BL, Income falls to $106.

Income Compensation for the SE

- Thus, according to Hicks and Slutsky, to **“compensate”** the consumer,
  - the income at the Pivoted BL will be reduced when Px falls.
  - the income at the Pivoted BL will be raised when Px rises.

Slutskian Method Example

LEARNING-BY-DOING EXERCISE 5.4

Finding Income and Substitution Effects Algebraically

In Learning-By-Doing Exercises 4.2 and 5.2, we met a consumer who purchases two goods, food and clothing. He has the utility function $U(x, y) = xy$, where x denotes the amount of food consumed and y the amount of clothing. His marginal utilities are $MU_x = y$ and $MU_y = x$. Now suppose that he has an income of \$72 per week and that the price of clothing is $P_y = \$1$ per unit. Suppose that the price of food is initially $P_{x_1} = \$9$ per unit and that the price subsequently falls to $P_{x_2} = \$4$ per unit.

Problem Find the numerical values of the income and substitution effects on food consumption, and graph the results.

Note: Use the “Slutskian” Substitution Effect

To find the solution:

Step 1: Find the demand equation

Step 2: Use the demand equation to find Bundles A and C.

Step 3: Find the compensated income and plug into the demand equation to find Bundle B.

LEARNING-BY-DOING EXERCISE 5.6

Income and Substitution Effects with a Quasilinear Utility Function

A college student who loves chocolate has a budget of \$10 per day, and out of that income she purchases chocolate x and a composite good y. The price of the composite good is \$1.

The quasilinear utility function $U(x, y) = 2\sqrt{x} + y$ represents the student's preferences. (See Chapter 3 for discussion of this kind of utility function.) For this utility function, $MU_x = 1/\sqrt{x}$ and $MU_y = 1$.

Problem

(a) Suppose the price of chocolate is initially \$0.50 per ounce. How many ounces of chocolate and how many units of the composite good are in the student's optimal consumption basket?

(b) Suppose the price of chocolate drops to \$0.20 per ounce. How many ounces of chocolate and how many units of the composite good are in the optimal consumption basket?

(c) What are the substitution and income effects that result from the decline in the price of chocolate? Illustrate these effects on a graph.

Ans(c): SE = 21, IE = 0 for the Hicksian Approach

Deriving Slutsky Equation

From TE = SE + IE, we can write it in a symbolic form.

$$\Delta X = \Delta X_{SE} + \Delta X_{IE}$$

…then..

$$\Delta X = \Delta X_{SE} - (\Delta X_{NIE})$$

where

$\Delta X = X_C(P_X' , I) - X_A(P_X , I)$

$\Delta X_{SE} = X_B(P_X' , I') - X_A(P_X , I)$ >>> Slutskian Method

$\Delta X_{IE} = X_C(P_X' , I) - X_B(P_X' , I')$

$\Delta X_{NIE} = - \Delta X_{IE} = X_B(P_X' , I') - X_C(P_X' , I)$ >>> "Negative" IE

Deriving Slutsky Equation

From $\Delta X = \Delta X_{SE} - \Delta X_{NIE}$,
we divide both sides with $\Delta Px = Px' - Px$.

$$\frac{\Delta X}{\Delta Px} = \frac{\Delta X_{SE}}{\Delta Px} - \frac{\Delta X_{NIE}}{\Delta Px}$$

In its derivative form,

$$\frac{\partial X}{\partial Px} = \frac{\partial X_{SE}}{\partial Px} - \frac{\partial X_{NIE}}{\partial Px}$$

Deriving Slutsky Equation

$$\frac{\partial X}{\partial Px} = \frac{\partial X_{SE}}{\partial Px} - \frac{\partial X_{NIE}}{\partial Px}$$

From I = PxX + PyY, we can write $\frac{\partial I}{\partial Px} = X$ or $\partial Px = \frac{\partial I}{X}$.

$$\mathbf{\frac{\partial X}{\partial Px} = \frac{\partial X_{SE}}{\partial Px} - \frac{\partial X_{NIE}}{\partial I} \cdot X}$$

This is called the “Slutsky Equation”.

Slutsky Equation

$$\frac{\partial \mathbf{X}}{\partial \mathbf{Px}} = \frac{\partial \mathbf{X}_{\mathbf{SE}}}{\partial \mathbf{Px}} - \frac{\partial \mathbf{X}_{\mathbf{NIE}}}{\partial \mathbf{I}} \cdot \mathbf{X}$$

$\frac{\partial X}{\partial Px}$ represents the TE.

$\frac{\partial X_{SE}}{\partial Px}$ represents the SE.

$-\frac{\partial X_{NIE}}{\partial I} \cdot X$ represents the IE.

We look at the "signs" of these derivatives when we interpret the Slutsky Equation because these signs tell us about the relationship between Px and X.

Slutsky Equation

$$\frac{\partial \mathbf{X}}{\partial \mathbf{Px}} = \frac{\partial \mathbf{X_{SE}}}{\partial \mathbf{Px}} - \frac{\partial \mathbf{X_{NIE}}}{\partial \mathbf{I}} \cdot \mathbf{X}$$

Note that $\frac{\partial X_{SE}}{\partial Px} < 0$, always. That is, SE is negative.

If X is a normal good, $\frac{\partial X_{NIE}}{\partial I} > 0$. But, IE is negative.

If X is an inferior good, $\frac{\partial X_{NIE}}{\partial I} < 0$. But, IE is positive.

“Negative” refers to the negative relationship b/w Px and X.

Slutsky Equation

$$\frac{\partial \mathbf{X}}{\partial \mathbf{Px}} = \frac{\partial \mathbf{X}_{\mathbf{SE}}}{\partial \mathbf{Px}} - \frac{\partial \mathbf{X}_{\mathbf{NIE}}}{\partial \mathbf{I}} \cdot \mathbf{X}$$

Conclusions

- If X is normal, SE and IE reinforce each other.
  - All effects are negative.
  - The Law of Demand holds, i.e. $\frac{\partial X}{\partial Px} < 0$.
- If X is inferior, SE and IE counteract each other.
  - SE is negative, but IE is positive.
  - The Law of Demand may not hold, e.g. Giffen good.