Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C) \rightarrow U'(C) = \frac{1}{C}$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score.....

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

$$\frac{1}{R_f} = \delta E\left[\frac{U'(C_1)}{U'(C_0)}\right]$$

$$\frac{1}{R_f} = \delta\left(\frac{C_1}{C_0}\right)$$

$$R_f = \frac{1}{\delta}\left(\frac{C_0}{C_1}\right)$$

∴ If Risk-free rate (R_f) is high, the consumption growth ($\frac{C_1}{C_0}$) will also high, meaning that the economy is boom. #

Score.....

ε

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$1 = E[m_{01}, R_f] \quad ; \quad m_{01} = \delta \frac{u'(c_1)}{u'(c_0)}$$

$$\varepsilon = \frac{R_f}{c_s/c_0} \cdot \frac{d(c_s/c_0)}{dR}$$

FOC : $\pi_s \cdot \delta\left(\frac{c_s}{c_0}\right) \cdot R_f = 1$

$$\pi_s \delta\left(-\frac{c_s}{c_0^2}\right) R_f \cdot d\left(\frac{c_s}{c_0}\right) + \pi_s \delta\left(\frac{c_s}{c_0}\right) \cdot dR_f = 0$$

$$\pi_s \delta\left(-\frac{c_s}{c_0^2}\right) R_f \cdot d\left(\frac{c_s}{c_0}\right) = -dR_f$$

$$\frac{\pi_s \delta\left(-\frac{c_s}{c_0^2}\right) R_f \cdot d\left(\frac{c_s}{c_0}\right)}{\pi_s \delta\left(\frac{c_s}{c_0}\right)} = -dR_f$$

$$-\frac{1}{c_0} R_f \cdot d\left(\frac{c_s}{c_0}\right) = -dR_f$$

$$\frac{R_s}{c_0} \cdot \frac{d(c_s/c_0)}{dR_f} = -1 \quad \#$$

∴ When the interest is decreasing, the individual's consumption level will decrease, which can be supported by the substitution effect; when interest rate rise, you will save less ($c_0 \uparrow$). Hence, c_1 is declining. #

Score.....

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$$P = E[m_{01} X_i]$$

$$= E\left[\frac{\delta \cdot u'(c_1)}{u'(c_0)} \cdot X_1\right]$$

∴ The meaning is that when c is low,

* u'(c) will be high, vice versa,

↓

* marginal utility

and expected payoff

Score.....

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

HJ - Bound,

$$\left| \frac{E[R_i] - R_f}{\sigma R_i} \right| \leq \frac{\sigma_{m_{01}}}{E[m_{01}]} = \sigma_{m_{01}} R_f$$