1. Given a linear system Ax = b where A is

$$\underline{\mathbf{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & C \end{bmatrix}$$

a) If C = 10, use **Gauss-Jordan method** to obtain an inverse of $\underline{\mathbf{A}}$ and solve $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$, give the solution in a vector form.

b) If C = 9 , obtain an inverse of $\underline{\mathbf{A}}$. Also determine det **A**

c) With C=9, solve $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$, give the solution in a vector form.

d) With C=9 and $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} 0 \\ 3 \\ D \end{bmatrix}$, find value of D that makes this linear system **inconsistent**.

2. An investment firm offers three stock portfolios: A, B, and C. The number of blocks of each type of stock in each of these portfolios is summarized in the following table:

		Portfolios		
		A	B	C
Risk	High	6	1	3
	Moderate	3	2	3
	Low	1	5	3

If a client wants to invest 35 blocks of high-risk stock (H), 22 blocks of moderate -risk stock (M) and 18 block of low-risk stock (L).

a) Write down the matrix equation for the above problem.
b) Use **row operations** to solve the matrix equation in a) and suggest a number of each portfolio needed.

3. Given that

$$\mathbf{B} = \begin{bmatrix} 2a & c & b & 0 \\ b & a+d & 0 & b \\ c & 0 & a+d & c \\ 0 & c & b & 2d \end{bmatrix}, \quad \underline{\mathbf{C}} = \begin{bmatrix} c & a+b & 0 & d \\ 0 & b & a & c+d \\ 0 & 0 & 0 & d+b \\ 0 & 0 & a+c & 2a \end{bmatrix}$$

$$\underline{D} = \begin{bmatrix} 2a & c & b & 2 & 0 \\ 2 & d & a+d & c & d \\ 0 & 0 & c & a & b \\ d & 0 & d & c & 0 \end{bmatrix}$$

a) Find $\det \underline{\mathbf{B}}$, $\det \underline{\mathbf{C}}$ and $\det \underline{\mathbf{D}}$
b) Show that if $\det \underline{\mathbf{B}} = 0$ then $a = -d$ or $ad = bc$
c) If $a = 1, b = 2, c = 1, d = 0$, find the $\det \underline{\mathbf{B}}$, $\det(2\underline{\mathbf{B}}^2)$ and $\det \underline{\mathbf{C}}$
d) With $\det \underline{\mathbf{B}}$ and $\det \underline{\mathbf{C}}$ in (c), find $\det \underline{\mathbf{A}}$ if $\det(\underline{\mathbf{A}}\underline{\mathbf{C}}^{-1})^{\mathrm{T}} = \det(\underline{\mathbf{C}}^{\frac{1}{3}}\underline{\mathbf{B}}^{-1})$

4. Find the volume under the surface $z = 2xy$ and above the rectangle bounded by the lines $x = 1$, $x = 2$, $y = 2$ and $y = 3$.

5. Output is given by

$$Q(K, L) = 50K^{3/5}L^{2/5}$$

Where K is the capital investment in units of \$1,000 and L is the size of the labour force measured in worker-hours. Suppose that monthly capital investment varies between \$10,000 and \$12,000, while monthly use of labour varies between 2,800 and 3,200 worker-hours. Find the average monthly output for the factory.