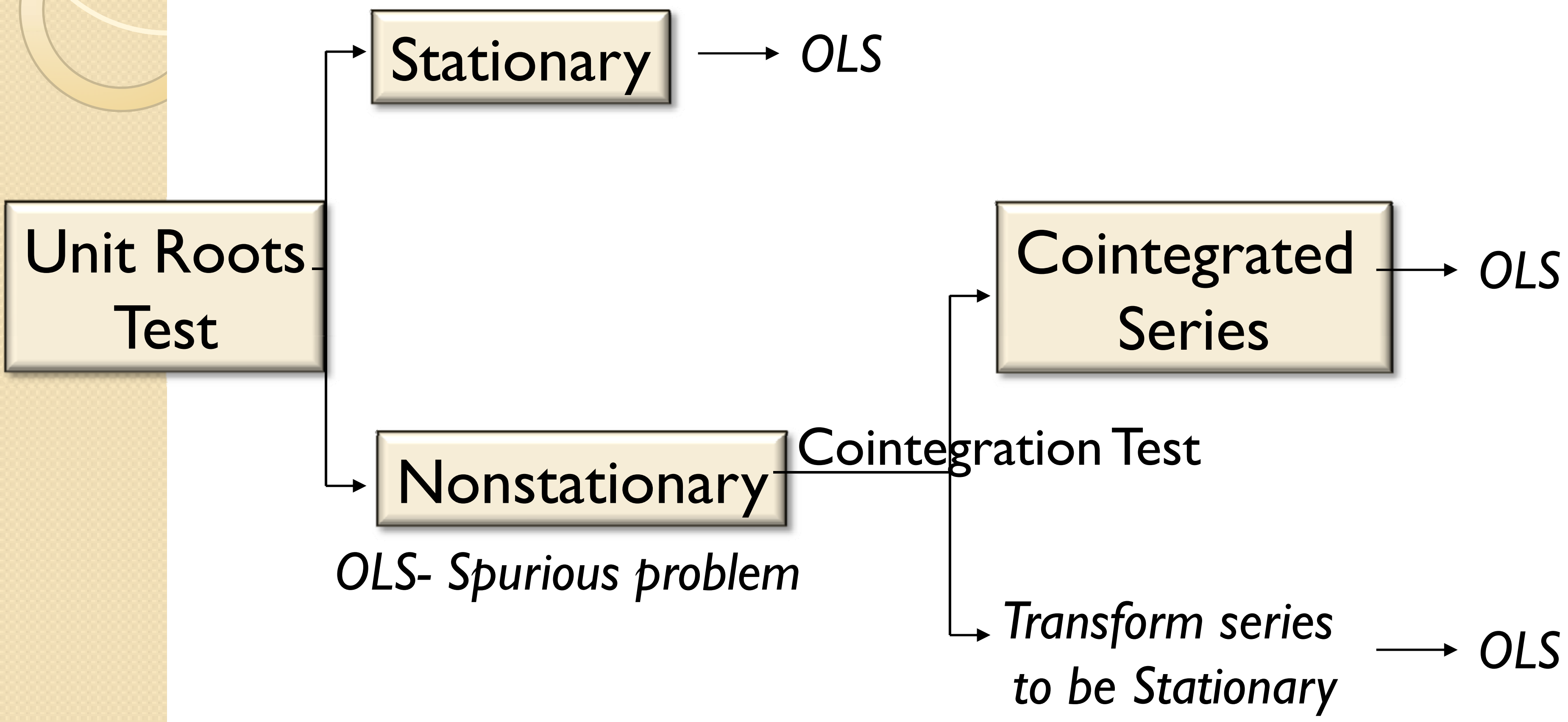
Time Series Econometrics

1. Basic Concept
2. Stochastic Processes
3. Unit Root Stochastic Processes
4. Trend Stationary (TS) and Difference Stationary (DS) Stochastic Processes
5. Integrated Stochastic Processes
6. Spurious Regression
7. Tests of Stationarity
8. Unit Root Test
9. Transforming Nonstationary Time Series
10. Cointegration

Basic Concept



Stochastic Processes

OLS assume that Xs must be nonstochastic process variables

Time-series is a sequence of random variables ordered in time.

<u>Stochastic Process</u>

The probability structure of a sequence of random variables is determined by the joint distribution of a stochastic process.

i.e. Y_t is stochastic series and white-noise process if $Y_t = u_t, \quad u_t \sim n.i.d.(0, \sigma^2)$

Stochastic Processes

Stochastic Process

- Stationary Process
- Nonstationary Process

Properties of Stationary Process

1. Mean of series must be stationary

$$E(Y_t) = \mu$$

2. Variance of series must be stationary

$$\text{var}(Y_t) = E(Y_t - \mu)^2 = \sigma^2$$

3. Covariance of Series must be stationary

$$\gamma_k = E\left[(Y_t - \mu)(Y_{t+k} - \mu)\right]$$

Nonstationary Processes

Random Walk without Drift

$$Y_t = Y_{t-1} + u_t$$

We can derive: $Y_1 = Y_0 + u_1$

$$Y_2 = Y_1 + u_2 = Y_0 + u_1 + u_2$$

Then, $Y_t = Y_0 + \sum u_t$

Therefore, $E(Y_t) = E\left(Y_0 + \sum u_t\right) = Y_0$

Then, $\text{var}(Y_t) = t\sigma^2$

and $(Y_t - Y_{t-1}) = \Delta Y_t = u_t$

Nonstationary Processes

Random Walk with Drift

$$Y_t = \delta + Y_{t-1} + u_t$$

where δ is drift parameter

Then, $(Y_t - Y_{t-1}) = \Delta Y_t = \delta + u_t$

Therefore, $E(Y_t) = Y_0 + t \cdot \delta$

and $\text{var}(Y_t) = t\sigma^2$

Unit Root Stochastic Processes

From random walk model:

$$Y_t = \rho Y_{t-1} + u_t \qquad -1 \le \rho \le 1$$

If $\rho = 1$, the model is random walk without drift.

The model then is known as unit root problem, or nonstationary.

However, if $|\rho| < 1$, the model is stationary.

These terms – nonstationary, random walk, and unit root – can be treated as synonymous.

Trend vs Difference Stationary

Deterministic vs Stochastic

Deterministic – if trend of time series is completely predictable and not variable.

Stochastic – if trend of time series is not predictable.

$$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

where u_t is a white noise error term.

t is time trend

Trend vs Difference Stationary

From: $$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

Pure random walk $\beta_1 = 0$, $\beta_2 = 0$, $\beta_3 = 1$

$$Y_t = Y_{t-1} + u_t \quad \text{and} \quad \Delta Y_t = (Y_t - Y_{t-1}) = u_t$$

Thus, random walk without drift is nonstationary while its first difference is stationary – called as difference stationary process (DSP).

Trend vs Difference Stationary

From: $$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

Random walk with drift $\beta_1 \neq 0$, $\beta_2 = 0$, $\beta_3 = 1$

$$Y_t = \beta_1 + Y_{t-1} + u_t \quad \text{and} \quad \Delta Y_t = (Y_t - Y_{t-1}) = \beta_1 + u_t$$

Thus, random walk with drift is nonstationary.

However, the series shows positive $(\beta_1 > 0)$ or negative $(\beta_1 < 0)$, then it is called stochastic trend.

Its first difference is stationary – also DSP process.

Trend vs Difference Stationary

From: $Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$

Deterministic Trend $\beta_1 \neq 0$, $\beta_2 \neq 0$, $\beta_3 = 0$

$$Y_t = \beta_1 + \beta_2 t + u_t$$

This series is trend stationary process (TSP) and mean equal $\bar{Y}_t = \beta_1 + \beta_2 t$, which is not constant, its variance is constant.

We can detrend the series by: $Y_t - \bar{Y}_t$

Trend vs Difference Stationary

From: $Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$

<u>Random walk with drift and deterministic trend</u>

$\beta_1 \neq 0$, $\beta_2 \neq 0$, $\beta_3 = 1$

$Y_t = \beta_1 + \beta_2 t + Y_{t-1} + u_t$ and $\Delta Y_t = \beta_1 + \beta_2 t + u_t$

Y_t is nonstationary process.

Trend vs Difference Stationary

From: $Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$

Deterministic trend with stationary AR(1) component $\beta_1 \neq 0$, $\beta_2 \neq 0$, $\beta_3 < 1$

$$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

Y_t is stationary around deterministic trend.

Integrated Stochastic Processes

Integrated Series

Nonstationary series can be integrated series if the series is differentiated one or more times, the resulting series will be stationary.

RWM without drift is nonstationary, but its first difference is stationary, the series is integrated of order *1* or *I(1)*.

If Y_t and its first difference (ΔY_t) are nonstationary, but its second difference is stationary, the series is integrated of order *2* or *I(2)*.

If Y_t is nonstationary, but its d^{th} difference is stationary, the series is integrated of order d or $I(d)$.

Integrated Stochastic Processes

Properties of Integrated Series

1. If $X_t \sim I(0)$ and $Y_t \sim I(1)$, then $Z_t = (X_t + Y_t) = I(1)$

   Linear combination of stationary and nonstationary time series is nonstationary.

2. If $X_t \sim I(d)$, then $Z_t = (a + bX_t) = I(d)$

   Linear combination of an $I(d)$ series is $I(d)$.

3. If $X_t \sim I(d_1)$ and $Y_t \sim I(d_2)$, then $Z_t = (aX_t + bY_t) = I(d_2)$ where $d_1 < d_2$.

4. If $X_t \sim I(d)$ and $Y_t \sim I(d)$, then $Z_t = (aX_t + bY_t) = I(d^*)$; d^* is generally equal to d, but in some case $d^* < d$.

Spurious Problem

If X_t and Y_t are uncorrelated nonstationary series, OLS estimated result of model

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

can lead to spurious problem.

Spurious regression has a high R^2 and t-statistics appear to be significant, but the results are without any theoretical meaning.

An $R^2 > DW$ is good rule of thumb to suspect that the estimated regression is spurious.

Unit Roots Test

Statistical test that test whether the series is stationary or nonstationary

- Dickey-Fuller (DF) Test
- Augmented Dickey-Fuller (ADF) Test
- Dickey-Fuller GLS (ERS)
- Phillips-Perron Test
- Kwiatkowski-Phillips-Schmidt-Shin
- Elliott-Rothenberg Stock Point-Optimal
- Ng-Perron

Unit Roots Test – DF Test

Suppose $Y_t = \alpha + \beta Y_{t-1} + u_t$

When, $t = 1$ then, $Y_1 = \alpha + \beta Y_0 + u_1$

$t = 2$, $Y_2 = \alpha + \beta\left(\alpha + \beta Y_0 + u_1\right) + u_2$

$$= \alpha\left(1+\beta\right) + \beta^2 Y_0 + \left(u_2 + \beta u_1\right)$$

Then,

$$Y_t = \alpha\left(1 + \beta + \beta^2 + \cdots + \beta^{t-1}\right) + \beta^t Y_0$$
$$+\left(u_t + \beta u_{t-1} + \beta^2 u_{t-2} + \cdots + \beta^{t-1} u_1\right)$$

For stationary condition, $\left|\beta\right| < 1$, then,

$$E(Y_t) = \mu = \frac{\alpha}{1-\beta}$$

Unit Roots Test – DF Test

If the series is stochastic trend process:

$$Y_t = \delta_0 + \delta_1 t + u_t \quad \text{and} \quad u_t = \beta u_{t-1} + \varepsilon_t$$

Then, $Y_t = \left[\delta_0 (1-\beta) + \beta\delta_1\right] + \delta_1 (1-\beta) t + \beta Y_{t-1} + \varepsilon_t$

DF test suggests to estimate the model:

$$Y_t - Y_{t-1} = \Delta Y_t = \underbrace{\left[\delta_0 (1-\beta) + \beta\delta_1\right]}_{\alpha_0} + \underbrace{\delta_1 (1-\beta)}_{\alpha_1} t + \gamma Y_{t-1} + \varepsilon_t$$

Where: $\gamma = \beta - 1$

Then, $\Delta Y_t = \alpha_0 + \alpha_1 t + \gamma Y_{t-1} + \varepsilon_t$

DF test -- τ (tau) statistic is t-test of γ using critical value from MacKinnon.

Unit Roots Test – ADF Test

DF test assumes *1st* order autocorrelation.

ADF test assumes higher order.

For example, *2nd* order autocorrelation:

$$Y_t = \delta_0 + \delta_1 t + u_t \quad \text{and} \quad u_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + \varepsilon_t$$

Then, the test equation:

$$\Delta Y_t = \underbrace{\left[\delta_0\left(1-\beta_1-\beta_2\right)+\left(\beta_1+\beta_2\right)\delta_1\right]}_{\text{Intercept}} + \underbrace{\delta_1\left(1-\beta_1-\beta_2\right)t}_{\text{Trend}} + \gamma Y_{t-1} + \underbrace{\beta_2 \Delta Y_{t-1}}_{\text{Lags}} + \varepsilon_t$$

Intercept

Trend

Lags

Unit Roots Test in Practice

Setting up the test:

- Testing Method
- Level of test
- Testing equation
- Optimal lags criteria

Unit-Test Process

1 Estimate $$\Delta y_t = \alpha_0 + \alpha_1 t + \gamma y_{t-1} + \sum_{i=1}^{m} \beta_i \Delta y_{t-i} + \varepsilon_t$$

Test $H_0 : \gamma = 0$ — No → No, there is no Unit Root

Yes ↓

Test $H_0 : \alpha_1 = 0$ — No → Yes, there is Unit Root.

Yes ↓

2 Estimate $$\Delta y_t = \alpha_0 + \gamma y_{t-1} + \sum_{i=1}^{m} \beta_i \Delta y_{t-i} + \varepsilon_t$$

Test $H_0 : \gamma = 0$ — No → No, there is no Unit Root

Yes ↓

Test $H_0 : \alpha_0 = 0$ — No → Yes, there is Unit Root.

Yes ↓

Unit-Test Process

3 Estimate $$\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{m} \beta_i \Delta y_{t-i} + \varepsilon_t$$

Test $H_0 : \gamma = 0$ —No→ No, there is no Unit Root

Yes ↓

Yes, there is Unit Root.

Cointegration

Cointegrated Time-series

If X_t and Y_t are nonstationary, but their linear combination $u_t = Y_t - \beta_1 - \beta_2 X_t$ is stationary.

Then, X_t and Y_t are cointegrated time-series.

Cointegration Test

Statistical test that tests whether the series are cointegrated or not.

- Augmented Engle-Granger (AEG) Test
- Multivariate (Johansen) Test

Cointegration and Error Correction Mechanism (ECM)

Cointegration Regression

Long-run relationship of cointegrated time series can be estimated using OLS.

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

Error Correction Mechanism (ECM)

ECM can be stated as:

$$\Delta Y_t = \alpha_0 + \alpha_1 \Delta X_t + \delta \hat{u}_{t-1} + \varepsilon_t$$

δ determines speed of adjustment to equilibrium and is expected to be negative and $-1 < \delta < 0$.