

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$(x_i - \bar{x})(y_i - \bar{y}) = x_i y_i$$

$$\begin{aligned} (63 - 77.625)(2.8 - 3.2125) &= 6.0328 \\ (72 - 77.625)(3.4 - 3.2125) &= -1.0547 \\ (78 - 77.625)(3 - 3.2125) &= -0.0797 \\ (81 - 77.625)(3.5 - 3.2125) &= 0.9703 \\ (87 - 77.625)(3.6 - 3.2125) &= 3.6328 \\ (75 - 77.625)(3 - 3.2125) &= 0.5578 \\ (75 - 77.625)(2.7 - 3.2125) &= 1.3453 \\ (90 - 77.625)(3.7 - 3.2125) &= 6.0328 \end{aligned}$$

$$\sum x_i y_i = 17.4374$$

$$n = 8$$

$$\sum x_i = 621$$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{621}{8}$$

$$= 77.625$$

$$\sum y_i = 25.7$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{25.7}{8}$$

$$= 3.2125$$

- 1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

$$\text{From OLS, } \hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$\hat{\beta}_2 = \frac{17.4374}{511.875} = 0.0341$$

$$\begin{aligned} \hat{\beta}_1 &= \bar{y} - \hat{\beta}_2 \bar{x} = 3.2125 - (0.0341) 77.625 \\ &= 0.5655 \end{aligned}$$

- 1.2 Find $\hat{y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$$

$$\hat{y}_i = 0.5625 + 0.0341 x_i$$

$$\hat{y}_1 = 0.5625 + 0.0341(63) = 2.7108, \hat{u}_1 = y_1 - \hat{y}_1 = 0.0892$$

$$\hat{y}_2 = 0.5625 + 0.0341(72) = 3.0177, \hat{u}_2 = y_2 - \hat{y}_2 = 0.3823$$

$$\hat{y}_3 = 0.5625 + 0.0341(78) = 3.2223, \hat{u}_3 = y_3 - \hat{y}_3 = -0.2223$$

$$\hat{y}_4 = 0.5625 + 0.0341(81) = 3.3246, \hat{u}_4 = y_4 - \hat{y}_4 = 0.1754$$

$$\hat{y}_5 = 0.5625 + 0.0341(87) = 3.5292, \hat{u}_5 = y_5 - \hat{y}_5 = 0.0708$$

$$\hat{y}_6 = 0.5625 + 0.0341(75) = 3.12, \hat{u}_6 = y_6 - \hat{y}_6 = -0.12$$

$$\hat{y}_7 = 0.5625 + 0.0341(75) = 3.12, \hat{u}_7 = y_7 - \hat{y}_7 = -0.42$$

$$\hat{y}_8 = 0.5625 + 0.0341(90) = 3.6315, \hat{u}_8 = y_8 - \hat{y}_8 = 0.0685$$

$$\sum_{i=1}^8 \hat{u}_i = 0.0239 \approx 0$$

1.3 Find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_1)$, and $\text{var}(\hat{\beta}_2)$

$$\begin{aligned}\text{var}(\hat{u}_i) &= \sigma^2 = \frac{\sum u_i^2}{n-2} \\ &= \frac{0.4348}{6} \\ &= 0.0725\end{aligned}$$

$$\begin{aligned}\text{var}(\hat{\beta}_1) &= \frac{\sum x_i^2}{n \sum x_i^2} \sigma^2 \\ &= 48.717 \times 0.0001 \\ &= 4.8717\end{aligned}$$

$$\begin{aligned}\text{var}(\hat{\beta}_2) &= \frac{\sigma^2}{\sum x_i^2} = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} \\ &= \frac{0.0725}{511.875} \\ &= 0.0001\end{aligned}$$

2. Data is listed in the table

X_i	Y_i	$(x_i - \bar{x})(y_i - \bar{y}) = x_i y_i$
10	0	$(10-20)(0-9.1) = 91$
12	2	$(12-20)(2-9.1) = 56.8$
14	5	$(14-20)(5-9.1) = 24.6$
16	6	$(16-20)(6-9.1) = 12.4$
18	7	$(18-20)(7-9.1) = 4.2$
22	10	$(22-20)(10-9.1) = 1.8$
24	10	$(24-20)(10-9.1) = 3.6$
26	15	$(26-20)(15-9.1) = 35.4$
28	16	$(28-20)(16-9.1) = 55.2$
30	20	$(30-20)(20-9.1) = 109$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{200}{10} = 20$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{91}{10} = 9.1$$

$$\sum x_i y_i = 384$$

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim \text{NIID}(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

$$\text{From OLS, } \hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{384}{440} = 0.8727$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} = 9.1 - (0.8727)(20) = -8.3545$$

2.2 Find the value of $\hat{y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$$

$$\hat{y}_i = -8.3545 + 0.8727x_i$$

$$\hat{y}_1 = -8.3545 + 0.8727(10) = 0.3725, \quad \hat{u}_1 = y_1 - \hat{y}_1 = -0.3725$$

$$\hat{y}_2 = -8.3545 + 0.8727(12) = 2.1179, \quad \hat{u}_2 = y_2 - \hat{y}_2 = -0.1179$$

$$\hat{y}_3 = -8.3545 + 0.8727(14) = 3.8633, \quad \hat{u}_3 = y_3 - \hat{y}_3 = 1.1367$$

$$\hat{y}_4 = -8.3545 + 0.8727(16) = 5.6087, \quad \hat{u}_4 = y_4 - \hat{y}_4 = 0.3913$$

$$\hat{y}_5 = -8.3545 + 0.8727(18) = 7.3541, \quad \hat{u}_5 = y_5 - \hat{y}_5 = -0.3541$$

$$\hat{y}_6 = -8.3545 + 0.8727(22) = 10.8449, \quad \hat{u}_6 = y_6 - \hat{y}_6 = 0.8449$$

$$\hat{y}_7 = -8.3545 + 0.8727(24) = 12.5903, \quad \hat{u}_7 = y_7 - \hat{y}_7 = -2.5903$$

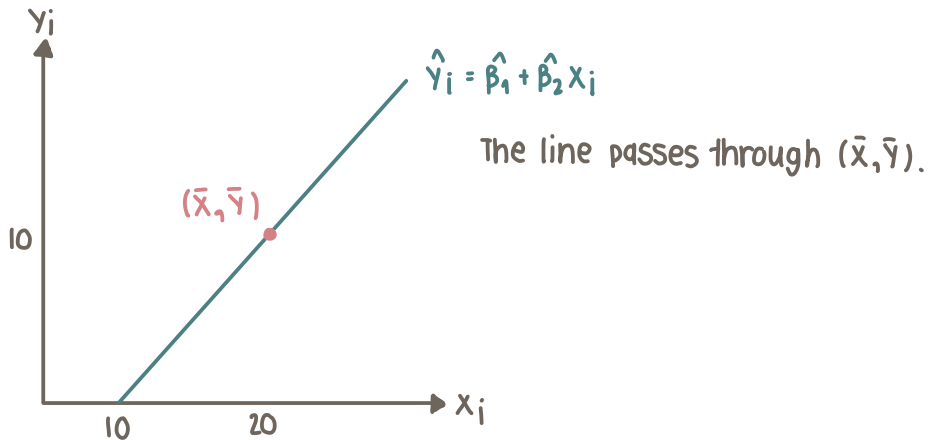
$$\hat{y}_8 = -8.3545 + 0.8727(26) = 14.3357, \quad \hat{u}_8 = y_8 - \hat{y}_8 = 0.6643$$

$$\hat{y}_9 = -8.3545 + 0.8727(28) = 16.0811, \quad \hat{u}_9 = y_9 - \hat{y}_9 = 0.0811$$

$$\hat{y}_{10} = -8.3545 + 0.8727(30) = 17.8265, \quad \hat{u}_{10} = y_{10} - \hat{y}_{10} = 2.1735$$

$$\sum \hat{u}_i = 0$$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{x}, \bar{y})$?



2.4 If $x_i = 18$, what is the predicted $\hat{y}$?

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$$

$$= -8.3545 + 0.8727(18) = 7.3541$$