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Lecture 2

20/08/2013

Chapter 4 Understanding what interest rate is

- Topic: Relationship between interest rates, bond price, returns.
  : What is yield to maturity (YTM) ?
  : Concept of present value (PV)

- interest rate = payment that borrower promise
  returns = actual amount of money that lender earns

- YTM = the most accurate measure of interest returns

- PV = the concept used to compare value of one debt instrument with another (different debt instrument have very different streams of cash payment to holders with different timing.

$1,000		$1,000
Value today	>	Value 1 year from now

(You would earn 30 $ if you deposit money at bank)
if i = 3%

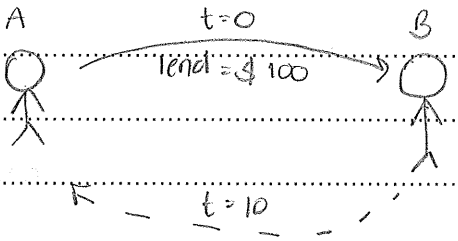
- There are 4 basic types of credit market instrument :
  1. A simple loan : loan repaid to lender at maturity date along with additional payment for interest ex commercial loan
  2. A fixed payment loan: loan repaid by making the same payment every period (ex every month) consisting of part of principal and interest ex installment loans (auto loans), mortgage etc.

3. A coupon bond : pay a fixed interest payment to bond owner (called coupon payment) every year until maturity date. The specified final amount (face value) is repaid at maturity date ex US Treasury bond and notes, corporate bonds.

4. A discount bond or a zero coupon bond : bought at a price below its face value (bought at discount). The face value is repaid at maturity date. There is no interest payment. ex US Treasury bill, US saving bond.

Note 1. Simple loan and discount bond make payment only at maturity date

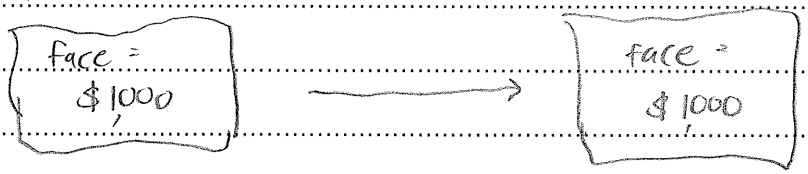
Simple loan :



return ⇒ $100 + interest payment 10% per year

return = $100(1+0.1)^{10}$

Discount bond :



t=0 buy at $900 t=1 sell at $1,000

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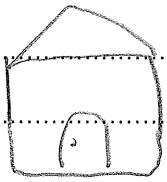
Note 2 : Fixed payment loan and coupon bond make payment periodically until maturity.

Fixed payment loan :



Ⓐ buying a house at t=0
borrow $1,000 from bank

Ⓐ pays bank $126 every year for 25 years

t=1 pay $126
t=2 $126
t=3 $126
⋮ ⋮
t=25 $126

A coupon bond :

face = $1000

A buy bond with face value $1,000
coupon rate 10% of face value
↳ coupon payment $100 per year
10 years maturity

t=0 Ⓐ buys bond and pay $1,000
t=1 Ⓐ receive $100
t=2 〃 $100
t=3 〃 $100
⋮ ⋮
t=10 〃 $100 + $1,000

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Present Value

1. Simple loan : principal, maturity

ex Ⓐ lend Ⓑ a simple loan of $100 for 1 year with interest of $10

→ interest for lending 1 year = $\frac{10}{100} = 0.1 = 10\%$

→ What happen if Ⓐ lend Ⓑ more than 1 year

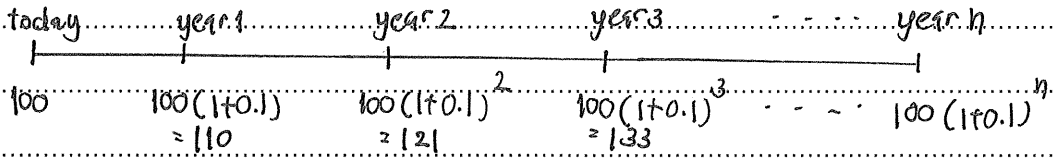
• end year 1 : value of loan is $100 (1+0.1) = $100(1+0.1) = 110 $

• " ⟶ " 2 : " ⟶ " $110(1+0.1) = $100(1+0.1)(1+0.1) = $121

$= \$100(1+0.1)^2$

• " ⟶ " 3 : " ⟶ " $121(1+0.1) = \$100(1+0.1)^3$

∴ At the end of n-year, $100 would turn into $100(1+i)^n$



The timeline also tell us that Ⓐ is just as happy having $100 today as having $110 a year from now, as having $121 two years from now, or $100(1+0.1)^n$ in n years from now.

From the timeline we can work backward from future amount to present

→ $133 three years from now is worth 100 today

$$133 = 100(1+0.1)^3$$

$$100 = \frac{133}{(1+0.1)^3}$$

present value (PV) = $\frac{CF}{(1+i)^n}$ — CF = future cash flow

calculating today's value of → dollar received in the future, "discounting the future"

- The concept of present value is useful in :

1) figure out today's value of credit market instrument at a given interest rate

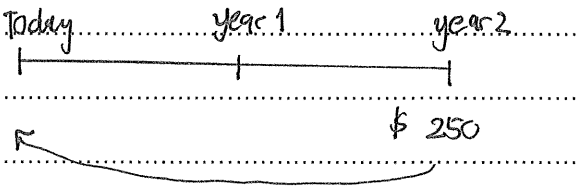
2) compare value of two or more instrument with different timing of their payments

ex What is the present value of $250 to be paid in 2 years if the interest rate is 15%.

$$PV = \frac{CF}{(1+i)^n}$$

$$= \frac{250}{(1+0.15)^2}$$

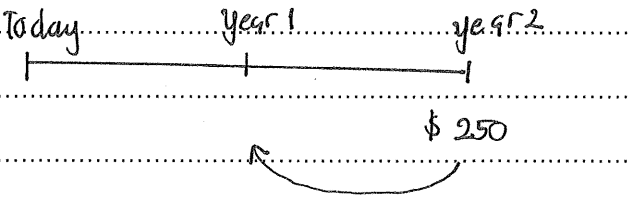
$$= \$189.04$$



What is the PV of $250 to be paid in 1 year if $i = 15\%$

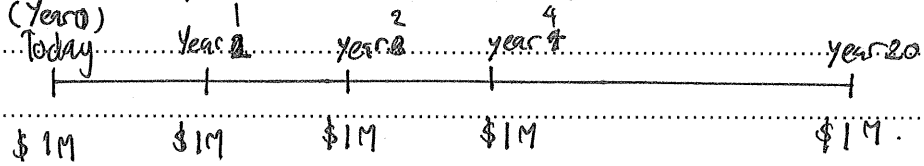
$$PV = \frac{250}{(1+0.15)^1}$$

$$= \$217.39$$



ex How much is a jackpot worth ?

Ⓐ won $20 million lottery, it promise a payment of $1 million for the next 20 years. Have you really won $20 million ?



If Interest rates is 10% ; $PV = \frac{CF}{(1+i)^n}$

Today : $1 M worth $1M

Year 2 : $1 M worth $\frac{CF}{(1+i)^n} = \frac{\$1M}{(1+0.1)^1} = \$909,090$

" 3 : " ⟶ " " $= \frac{\$1M}{(1+0.1)^2} = \$826,446$

⋮

" 20 : " ⟶ " " $= \frac{\$1M}{(1+0.1)^{19}} = \$163,507$

To sum up, present value of $20 million becomes $9.4 million

Yield to maturity (YTM)

YTM is the interest rates that equates PV of cashflow payment received from a debt instrument with its value today.

YTM is the most accurate measure of interest rates.

① Simple loan

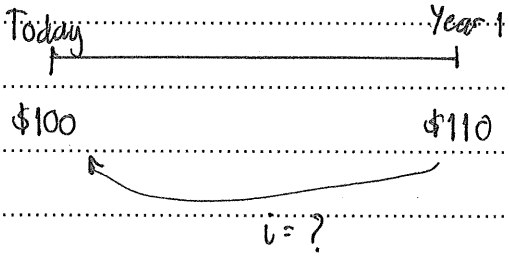
ex Ⓐ borrow $100 from Ⓑ. Next year Ⓑ wants $110 back from Ⓐ. What is YTM on this loan?

$$PV = \frac{CF}{(1+i)^n}$$

CF: cashflow in 1 year; PV: amount borrowed; n: number of year

$$100 = \frac{110}{(1+i)^1} \quad \therefore i = \frac{110}{100} - 1 = 0.1 = 10\%$$

$\therefore$ YTM = 10%



For simple loan, simple interest rates equals YTM

② Fixed payment loan : same cash flow payment every period.

ex loan = $1000

yearly payment = $126 for the next 25 years. find YTM = ?

loan value → $PV = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \dots + \frac{CF}{(1+i)^{25}}$ ← fixed yearly payment; ↑ number of years until maturity

$$\$1000 = \frac{\$126}{(1+i)} + \frac{\$126}{(1+i)^2} + \frac{\$126}{(1+i)^3} + \dots + \frac{\$126}{(1+i)^{25}}$$

Solve for i

ex ⓐ purchase a new home and need $100,000 mortgage.

interest rate = 7%.

payoff loan in 20 years

yearly payment to bank = ?

$$PV = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \dots + \frac{CF}{(1+i)^n}$$

$$100{,}000 = \frac{CF}{1.07} + \frac{CF}{(1.07)^2} + \dots + \frac{CF}{(1.07)^{20}}$$

Solve for CF ~ $9,439.29

③ coupon bond

PV of bond = sum of all coupon payment plus present value of final payment of face value of bond.

ex face value of bond = $1000
maturity = 10 years
yearly coupon payment = $100 (10% coupon rate), find price = ?

price of coupon bond (P) ← yearly coupon payment, face value of bond

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \dots + \frac{C}{(1+i)^{10}} + \frac{F}{(1+i)^{10}}$$

↖ year to maturity date

$$= \frac{100}{1+i} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + \dots + \frac{100}{(1+i)^{10}} + \frac{1000}{(1+i)^{10}}$$

↑ YTM = 10% = coupon rate.
> 10%

ex face value = $1,000
maturity = 8 years
coupon rate = 10%
YTM = 12.25%

$$P = \frac{100}{1.1225} + \frac{100}{(1.1225)^2} + \dots + \frac{100}{(1.1225)^8} + \frac{1,000}{(1.1225)^8}$$

bond price = $889.2

Note : when YTM > coupon rate

bond price < face value

Ex 10%. coupon bond

Face value = $1,000

10 years maturity

price of bond varies

1) $P = \$1,200$; $1200 = \frac{100}{(1+i)} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + \ldots + \frac{100}{(1+i)^{10}} + \frac{1000}{(1+i)^{10}}$

↳ YTM = i = 7.13%

2) $P = \$1,100$; $1100 = \frac{100}{(1+i)} + \frac{100}{(1+i)^2} + \frac{100}{(1+i)^3} + \ldots + \frac{100}{(1+i)^{10}} + \frac{1000}{(1+i)^{10}}$

↳ YTM = i = 8.48%.

3) $P = \$1,000$ → YTM = i = 10%.

4) $P = \$900$ → i = 11.75%.

5) $P = \$800$ → i = 13.81%

Note: 1) When coupon bond priced at face value

YTM = coupon rate

2) Price of bond & YTM are negatively related

YTM ↑, price bond ↓

YTM↑ mplies that future coupon payment & final payment are worthless when discount back to present value, hence price & bond must be lower.

4. Discount bond

ex 1 year US Treasury-bill

price of current purchase = $900

In 1 year time, pay face value of $1,000

$$PV = \frac{FV}{(1+i)^1}$$

$$900 = \frac{1000}{1+i}$$

$$i = \frac{1000}{900} - 1 = 0.111 = 11.1\%$$

Distinction between interest rates and returns

rate of return is defined as payment to the owner plus change in its value, expressed as a fraction of its purchase price

ex A coupon bond

face value = $1000

coupon rate = 10%

price = $1000

time to maturity = 1 year

sold for $1,200 after 1 year

$\therefore$ return on bond $= \frac{C}{P_t} + \left(\frac{P_{t+1m} - P_t}{P_t}\right)$

yield: $\frac{100}{1000}$ + capital gain: $\left(\frac{1200 - 1000}{1000}\right)$

$= \frac{300}{1000} = 0.3 = 30\%$

$R = i + g.$

Real vs Nominal Interest rates

Real interest rate is nominal interest rate adjusted for change in price level. (it tells real purchasing power)

$$i = r + \pi^e$$

i: nominal interest rates
r: real interest rate
π^e: expected change in price level.

ex (A) make 1 year simple loan with 5% interest rates
expect the price level rise by 3%.

↳ real interest return = 5 - 3 = 2%.

ex nominal interest rates = 8%.
expected inflation rate = 10%
↳ ∴ real interest rates = 8 - 10 = -2% ⇒ real cost of borrowing
⇒ real return to lender
∴ you will be able to buy 2% fewer goods at the end of the year.

↳ when interest rate is low, greater incentives to borrow but less incentive to lend.