

# Topic 4 : Capital Asset Pricing Model and Arbitrage Pricing Theory (Part 2)

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Copeland, Thomas E. and J. Fred Weston, Financial Theory and Corporate Policy (4th ed), Addison-Wesley, 2005: Ch6 (pp 147 -157)

Federic Mishkin, The Economics of Money, Banking and Financial Markets (Appendix to Chapter 5, available in the internet):

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# The Arbitrage Pricing Theory

- CAPM:  $R_i = (R_f - \beta_i R_f) + \beta_i R_m + \epsilon_i$ ,  $R_i = \beta_0 + \beta_i R_m + \epsilon_i$ 
  - only one source of systematic risk, unsystematic risk can be eliminated through diversification
- The APT: several sources of risk that cannot be eliminated through diversification
- The sources of risk : inflation, aggregate output, .. etc.
- APT:  $R_i = \beta_0^i + \beta_1^i (\text{Return on Factor 1}) + \beta_2^i (\text{Return on Factor 2}) + \dots + \beta_k^i (\text{Return on Factor k}) + \epsilon^i$
- Assets with the same values of  $\beta_j$  for all factors  $j$  must have the same rate of returns.
- If there is only one factor and that factor is  $R_m$ , the APT is the same as the CAPM.
- The APT is more general than the CAPM

## The Arbitrage Pricing Theory : arbitrage opportunity

- Arbitrage : an arbitrage opportunity arises when an investor can construct a zero investment portfolio that will yield a sure profit (risk-free)

- Example :

	$S_1$	$S_2$	<i>Price</i>
● $A_1$	1	0	0.2
$A_2$	0	1	0.1

- If there exists a security C pay 2 \$ when  $S_1$  occurs and it is priced at \$ 0.5, we can construct an arbitrage portfolio; selling at a high price and buying at a low price.
- In equilibrium, no arbitrage portfolio exists.
- Law of one price

- Example :  $E(R_a) = 0.08 + 0.6F_1$  ,  $E(R_b) = 0.02 - 0.2F_1$ .
  - Determine the portfolio weights you need to place on a and b in order to construct a portfolio which has no risk ( $\beta_1 = \dots$ ).
  - If the risk free rate is equal to 0.01 (you can lend or borrow at 1% interest rate), can you make an arbitrage profit?

- At equilibrium, there must be no arbitrage opportunity.
- From the last example,  $E(R_a) = 0.08 + 0.6F_1$  ,  
 $E(R_b) = 0.02 - 0.2F_1$ .
  - Suppose there is asset C,  $E(R_C) = 0.10 + 0.6F_1$ .
  - Can you make an arbitrage profit?
  - Buy low and sell high.
  - Buy ..... and short-sell .....
  - get a profit of .....=0.02 = 2%, regardless of the value of  $F_1$ .

- Short-sell : The sale of shares not owned by the investor but borrowed through a broker and later repurchased to preplace the loan. Profit is earned if the initial sale is at higher price than the repurchase price.
- $E(R_i) = \beta_0^i + \beta_1^i F_1 + \beta_2^i F_2 + \dots + \beta_k^i F_K$
- Assets with the same values of  $\beta_j$  for all factors  $j$  must have the same rate of returns.
- Example :  $E(R_a) = 0.08 + 0.6F_1$  ,  $E(R_b) = 0.02 - 0.2F_1$ .
  - $R_f$  = the rate of return of the portfolio which has  $\beta_1 = 0$ .
  - Determine the portfolio weights you need to place on a and b in order to construct a portfolio which has  $\beta_1 = 1$ .
  - $R_m = 0.11 + F_1$
  - Determine the rate of returns of portiolio i which has 0, 0.6, 0.2, 1, 0.5, 2.

- $E(R_i) = R_f + (E(R_m) - R_f)\beta_1^i$
- $E(R_i) = \dots\dots\dots + \dots\dots\dots$

- $\beta = 0.6 \Rightarrow$   
 $E(R_i) = 0.035 + 0.6(0.11 + F_1 - 0.035) = 0.08 + 0.6F_1$

- $\beta = -0.2 \Rightarrow$   
 $E(R_i) = 0.035 - 0.2(0.11 + F_1 - 0.035) = 0.02 - 0.2F_1$

- CAPM :  $R_i = R_f + \beta(\text{Market Risk Premium})$

- APT :  
 $E(R_i) = \beta_0^i + \beta_1^i(\text{Factor 1}) + \beta_2^i(\text{Factor 2}) + \dots + \beta_k^i(\text{Factor k})$

- $E(R_i) = R_f + (\lambda_1 - R_f)\beta_1^i + (\lambda_2 - R_f)\beta_2^i + \dots + (\lambda_k - R_f)\beta_k^i ;$

- $\lambda_j =$  the expected rate of return on portfolio with unit sensitivity to the  $j$  the factor and zero sensitivity to all other factors.

- At equilibrium, there is no arbitrage opportunity.

- Examples: 2 Factors

$$E(R_a) = 0.10F_1 - 0.5F_2$$

$$E(R_b) = 0.08 + 2F_1 + F_2$$

$$E(R_c) = 0.05 + 0.5F_1 + 0.5F_2$$

$$E(R_p) = w_a E(R_a) + w_b E(R_b) + w_c E(R_c)$$

- Determine the portfolio weights you need to place on a and b in order to construct a portfolio which has no risk

$$0.10w_a + 2w_b + 0.5w_c = 0$$

$$-0.5w_a + 1w_b + 0.5w_c = 0$$

$$w_a = \frac{5}{13}, w_b = -\frac{3}{13}, w_c = \frac{11}{13}$$

- Determine the rate of return on portfolio which has  $\beta_1 = 1$  and  $\beta_2 = 0$ .
- Determine the rate of return on portfolio which has  $\beta_1 = 0$  and  $\beta_2 = 1$
- $E(R_i) = R_f + (\lambda_1 - R_f)\beta_1^i + (\lambda_2 - R_f)\beta_2^i$ ;  $\lambda_j$  = the expected rate of return on portfolio which has  $\beta_j = 1$  and other betas equal to zero.