EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i	$y-\bar{y}$	$x-\bar{x}$
1	2.8	63	-0.4125	-14.625
2	3.4	72	0.1875	5.625
3	3	78	-0.2125	0.375
4	3.5	81	0.2875	3.375
5	3.6	87	0.3875	9.375
6	3.0	75	-0.2125	-2.625
7	2.7	75	-0.5125	-2.625
8	3.7	90	-0.4875	12.375

$$\bar{y} = \frac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8}$$

$= 3.2125$

$$\bar{x} = \frac{63+72+78+81+87+75+75+90}{8}$$

$= 77.625$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{17.4375}{511.875} = 0.0341 \#$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$= 3.2125 - (0.0341)(77.625)$

$= 0.5681$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$	$\hat{u}_i = y_i - \hat{y}_i$
$\hat{y}_1 = 2.7143$	$\hat{u}_1 = 0.0857$
$\hat{y}_2 = 3.0209$	$\hat{u}_2 = 0.3791$
$\hat{y}_3 = 3.1153$	$\hat{u}_3 = -0.2253$
$\hat{y}_4 = 3.3275$	$\hat{u}_4 = 0.1725$
$\hat{y}_5 = 3.5319$	$\hat{u}_5 = 0.0681$
$\hat{y}_6 = 3.1231$	$\hat{u}_6 = -0.1231$
$\hat{y}_7 = 3.1231$	$\hat{u}_7 = -0.4231$
$\hat{y}_8 = 3.6341$	$\hat{u}_8 = 0.0659$

It's true that $\sum_{i=0}^{8} = 0$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$Var(\hat{\beta}_0) = \frac{\sum x_i^2}{n\sum(x_i-\bar{x})^2}\sigma^2$$

2. Data is listed in the table

X_i	Y_i	$y_i-\bar{y}$	$x_i-\bar{x}$
10	0	-9.1	-10
12	2	-7.1	-8
14	5	-4.1	-6
16	6	-3.1	-4
18	7	-2.1	-2
22	10	0.9	2
24	10	0.9	4
26	15	5.9	6
28	16	6.9	8
30	20	10.9	10

$$\bar{y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$$

$$\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = \frac{200}{10} = 20$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

find $\hat{\beta}_0$ and $\hat{\beta}_1$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

$$= \frac{91+56.9+24.6+12.4+4.2+1.8+3.6+35.4+55.2+109}{100+64+36+16+4+4+16+36+64+100}$$

$$= \frac{394}{440} = 0.8955 \ \#$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$$

$$\hat{\beta}_0 = 9.1 - (0.8955\times 20)$$

$$= -8.81 \ \#$$

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$ $\hat{u}_i = y_i - \hat{y}_i$

2.2 Find the value of $\widehat{Y}_\iota$ and $\widehat{u}_\iota$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

0.1455	-0.1455
1.9364	0.0636
3.7273	1.2727
5.5182	0.4818
9.3091	-0.3091
10.8909	-0.8909
12.6818	-2.6818
14.4729	0.5273
11.2636	-0.2636
18.0546	1.9455

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$? (20, 9.1)

$y = 1.5x + c$

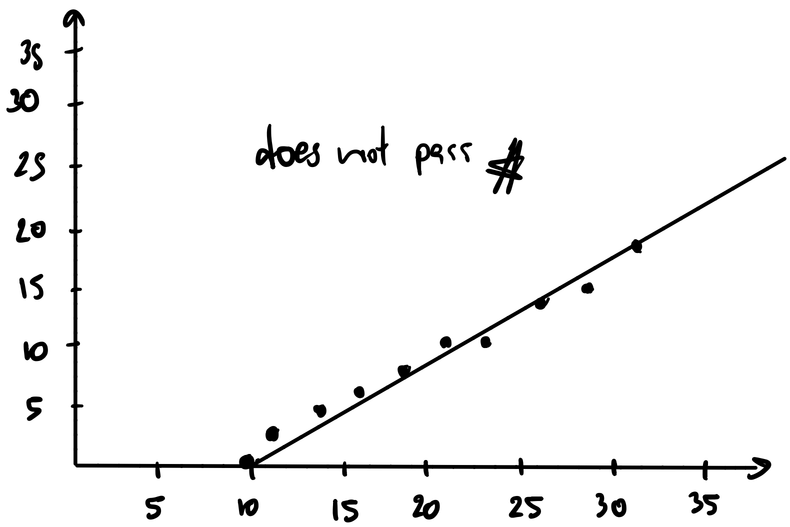
$5 = 1.5 \times 14 + c$

$c = 16$

$y = 1.5x - 16$

$9.1 = 1.5(20) - 16$

$\neq$



2.4 If X_i =16, what is the predicted Y?

$E(y|x = 16) = 5.5182$

2.5 Find $var(\widehat{u_\iota})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

unbiased $E(\hat{\beta_1}) = \beta_1$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$u_i = y_i - \beta_1 x_i$

denoted the summation square as $f(\beta_0, \beta_1) = \sum_{i=1}^{n} (y_i - \beta_1 x_i)^2$

F.O.C. derivative w.r.t. $\beta_1 = 0$

$0 = 2 \sum_{i=1}^{n} (y_i - \beta_1 x_i)$