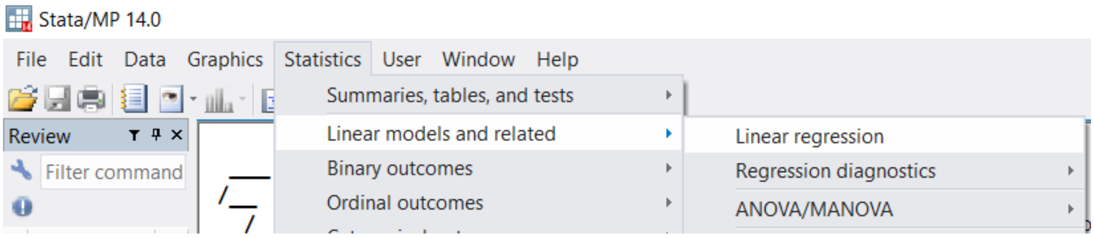
Introduction to the Regression Model

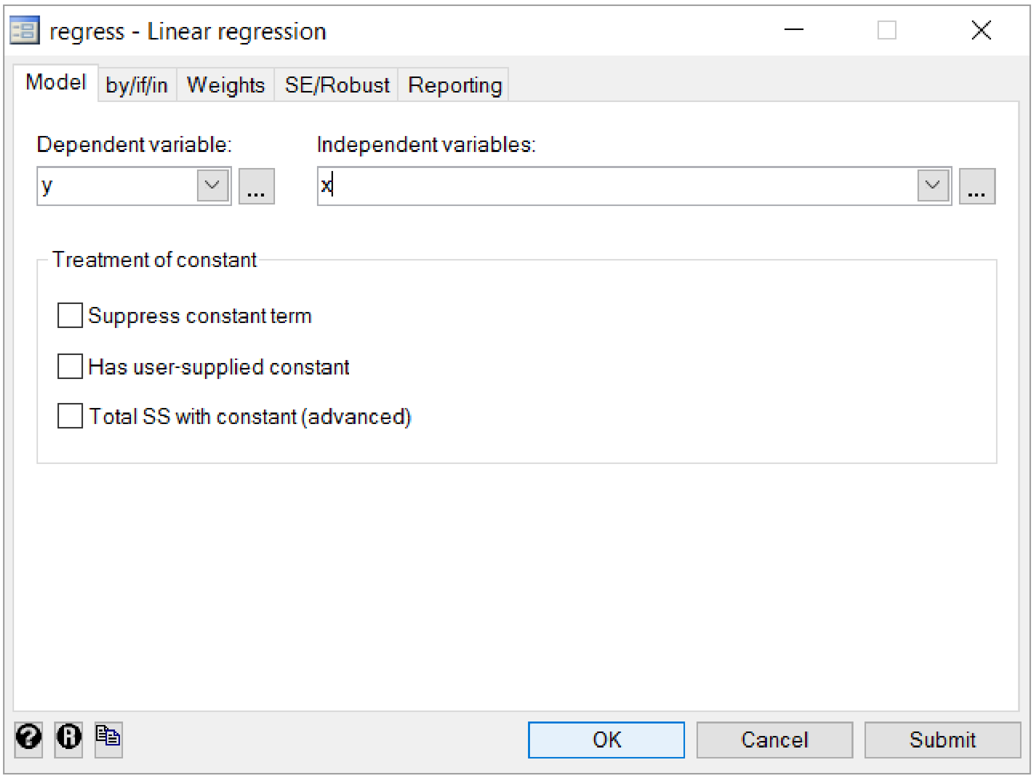
<u>Simple Regression Model</u>

CAPM model: $$Y_t = \beta_0 + \beta_1 X_{1t} + u_t$$

In this example, we estimate the least-squares regression of Y_i on X_i. To run a regression, from menu bar go to Statistics, choose Linear models and related, select Liner regression.



From regress – Linear regression window, specify dependent and independent variables in Dependent variable: and Independent variables: boxes, then, click OK.



The results will be as follows:

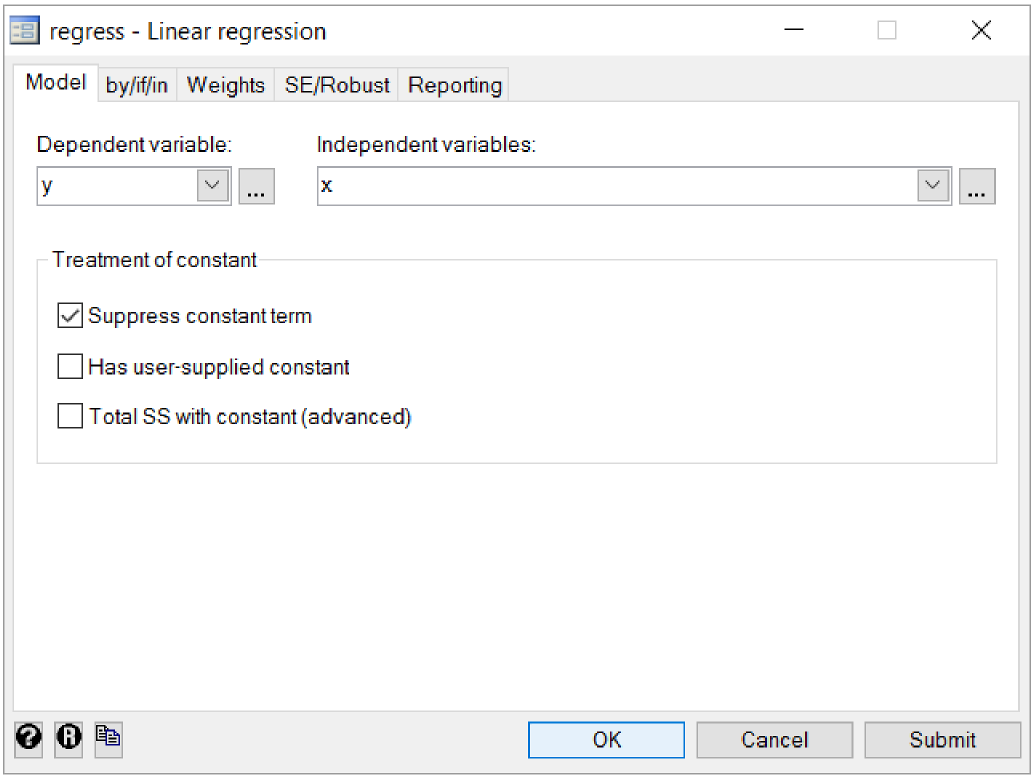
```
. reg y x1

      Source |       SS           df       MS      Number of obs   =       120
-------------+----------------------------------   F(1, 118)       =     64.86
       Model |  247.518473         1  247.518473   Prob > F        =    0.0000
    Residual |   450.31445       118  3.81622416   R-squared       =    0.3547
-------------+----------------------------------   Adj R-squared   =    0.3492
       Total |  697.832923       119  5.86414221   Root MSE        =    1.9535

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .9605023   .1192646     8.05   0.000     .7243259    1.196679
       _cons |    .380914   .1783442     2.14   0.035     .0277441     .734084
------------------------------------------------------------------------------

.est store withcon
```

To run the model without intercept term, check Suppress constant term box.




```
. reg y x1, nocon

      Source |       SS           df       MS      Number of obs   =       120
-------------+----------------------------------   F(1, 119)       =     63.39
       Model |  249.166211         1  249.166211   Prob > F        =    0.0000
    Residual |  467.723299       119  3.93044789   R-squared       =    0.3476
-------------+----------------------------------   Adj R-squared   =    0.3421
       Total |   716.88951       120  5.97407925   Root MSE        =    1.9825

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .9636218   .1210272     7.96   0.000     .7239758    1.203268
------------------------------------------------------------------------------

. est store wocon
```

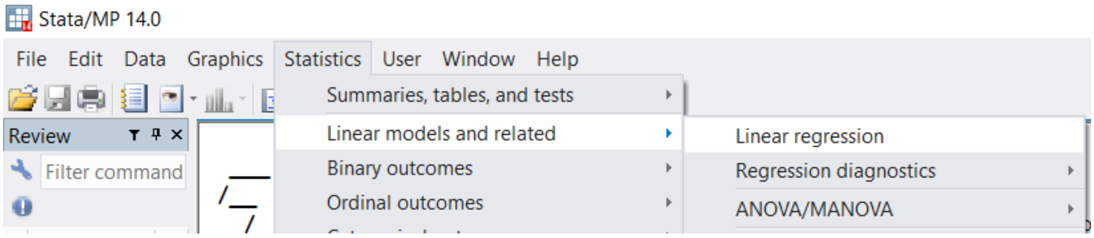
```
. est table withcon wocon, star(0.1 0.05 0.01) stat(N rss F r2 r2_a)

----------------------------------------------
    Variable |    withcon          wocon
-------------+--------------------------------
          x1 |  .96050229***     .96362177***
       _cons |  .38091403**
-------------+--------------------------------
           N |        120              120
         rss |  450.31445         467.7233
           F |  64.859521        63.393847
          r2 |  .35469589        .34756571
        r2_a |  .34922722        .34208307
----------------------------------------------
           legend: * p<.1; ** p<.05; *** p<.01
```

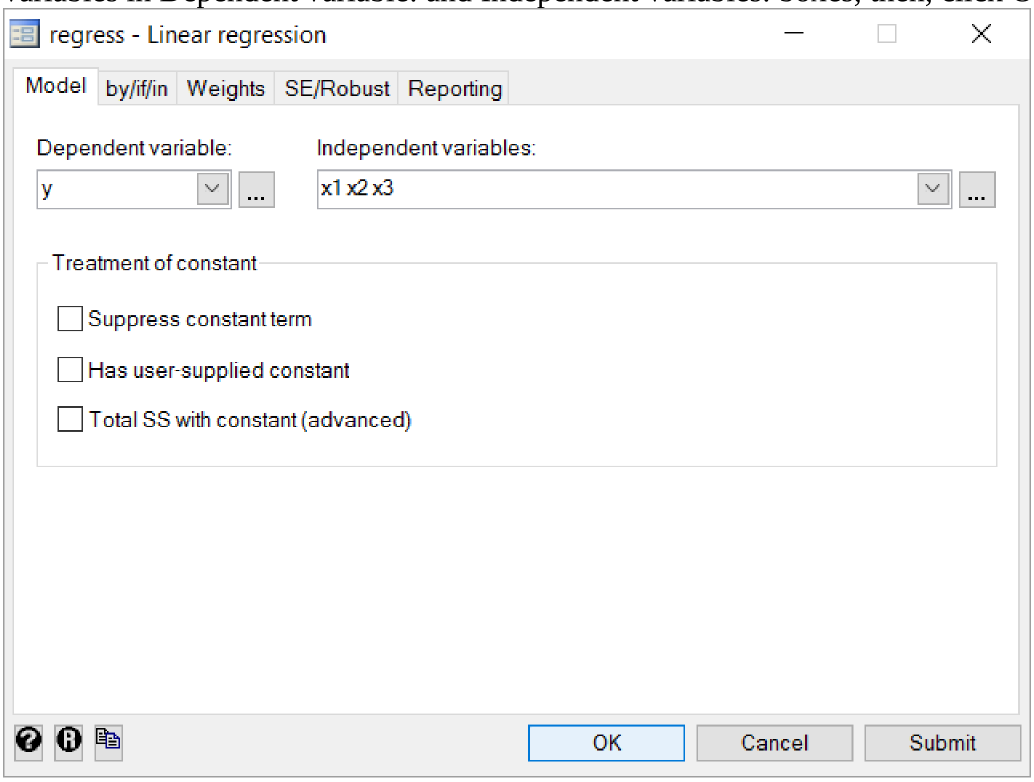
<u>Multiple Regression Model</u>

Fama-French model: $Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

To run a regression, from menu bar go to Statistics, choose Linear models and related, select Liner regression.



From regress – Linear regression window, specify dependent and independent variables in Dependent variable: and Independent variables: boxes, then, click OK.



The results will be as follows:

```
. reg y x1 x2 x3

      Source |       SS           df       MS      Number of obs   =       120
-------------+----------------------------------   F(3, 116)       =     69.03
       Model |  447.276407         3  149.092136   Prob > F        =    0.0000
    Residual |  250.556516       116  2.15996997   R-squared       =    0.6410
-------------+----------------------------------   Adj R-squared   =    0.6317
       Total |  697.832923       119  5.86414221   Root MSE        =    1.4697

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   .8614489   .0906645     9.50   0.000     .6818765    1.041021
          x2 |   .4118276   .0581692     7.08   0.000     .2966163     .527039
          x3 |   .2799795   .0419203     6.68   0.000      .196951     .363008
       _cons |   .1439872   .1368248     1.05   0.295    -.1270116    .4149859
------------------------------------------------------------------------------
```

Hypotheses Testing

To test whether CAPM or FF model is more appropriated or should we eliminate x_2 and x_3 out of the model, or H_0: $\beta_2 = \beta_3 = 0$.

```
. test x2 x3

 ( 1)  x2 = 0
 ( 2)  x3 = 0

       F(  2,   116) =   46.24
            Prob > F =    0.0000
```

Dummy Variable Regression Models

ANOVA Model

In the study of seasonal effect (Monday and Friday effects), ANOVA model is estimated using both models with and without intercept term:

Model with Intercept term: $Y_t = \beta_1 + \beta_2 D_{2t} + \beta_3 D_{3t} + \beta_4 D_{4t} + \beta_5 D_{5t} + u_t$

Model without Intercept term: $Y_t = \beta_1 D_{1t} + \beta_2 D_{2t} + \beta_3 D_{3t} + \beta_4 D_{4t} + \beta_5 D_{5t} + u_t$

where: Y_t = Return on asset at time t.
D_{1t} = 1 on Monday and = 0 otherwise.
D_{2t} = 1 on Tuesday and = 0 otherwise.
D_{3t} = 1 on Wednesday and = 0 otherwise.
D_{4t} = 1 on Thursday and = 0 otherwise.
D_{5t} = 1 on Friday and = 0 otherwise.

```
. reg y d2 d3 d4 d5

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  4,   990) =    7.60
       Model |  .00681682     4  .001704205           Prob > F      =  0.0000
    Residual | .221894717   990  .000224136           R-squared     =  0.0298
-------------+------------------------------           Adj R-squared =  0.0259
       Total | .228711537   994  .000230092           Root MSE      =  .01497

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          d2 |   .0044637   .0015009     2.97   0.003     .0015184    .0074089
          d3 |   .0044351   .0015009     2.96   0.003     .0014898    .0073803
          d4 |   .0040519   .0015009     2.70   0.007     .0011067    .0069972
          d5 |   .0082592   .0015009     5.50   0.000      .005314    .0112045
       _cons |  -.0040211   .0010613    -3.79   0.000    -.0061037   -.0019385
------------------------------------------------------------------------------

. reg y d1 d2 d3 d4 d5, noconstant

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  5,   990) =    6.13
       Model | .006865358     5  .001373072           Prob > F      =  0.0000
    Residual | .221894717   990  .000224136           R-squared     =  0.0300
-------------+------------------------------           Adj R-squared =  0.0251
       Total | .228760075   995   .00022991           Root MSE      =  .01497

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          d1 |  -.0040211   .0010613    -3.79   0.000    -.0061037   -.0019385
          d2 |   .0004426   .0010613     0.42   0.677    -.0016401    .0025252
          d3 |    .000414   .0010613     0.39   0.697    -.0016687    .0024966
          d4 |   .0000308   .0010613     0.03   0.977    -.0020518    .0021134
          d5 |   .0042381   .0010613     3.99   0.000     .0021555    .0063207
------------------------------------------------------------------------------
```

Interpretations of the estimated results of the two models are as follows:

Model	With intercept term	Without intercept
Monday	$\hat{Y}_t = \hat{\beta}_1$ = -0.004021	$\hat{Y}_t = \hat{\beta}_1$ = -0.004021
Tuesday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_2$ = -0.004021+0.004464 = 0.000443	$\hat{Y}_t = \hat{\beta}_2$ = 0.000443
Wednesday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_3$ = -0.004021+0.004435 = 0.000414	$\hat{Y}_t = \hat{\beta}_3$ = 0.000414
Thursday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_4$ = -0.004021+0.004052 = 0.000031	$\hat{Y}_t = \hat{\beta}_4$ = 0.000031
Friday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_5$ = -0.004021+0.008259 = 0.004238	$\hat{Y}_t = \hat{\beta}_5$ = 0.004238

For the purpose of testing only Monday and Friday effect, the model can be stated as:

Model to test Monday and Friday effect: $Y_t = \beta_0 + \beta_1 D_{1t} + \beta_5 D_{5t} + u_t$

```
. reg y d1 d5

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  2,   992) =   15.19
       Model |  .006795783     2  .003397892           Prob > F      =  0.0000
    Residual |  .221915754   992  .000223705           R-squared     =  0.0297
-------------+------------------------------           Adj R-squared =  0.0278
       Total |  .228711537   994  .000230092           Root MSE      =  .01496

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          d1 |  -.0043169   .0012243    -3.53   0.000    -.0067194   -.0019144
          d5 |   .0039423   .0012243     3.22   0.001     .0015399    .0063448
       _cons |   .0002958   .0006121     0.48   0.629    -.0009055     .001497
------------------------------------------------------------------------------
```

Interpretation of the estimated result of the model is as follow:

Model	
Tuesday **Wednesday** **Thursday**	$\hat{Y}_t = \hat{\beta}_0$ = 0.000296
Monday	$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1$ = 0.000296 - 0.004317 = -0.004021
Friday	$\hat{Y}_t = \hat{\beta}_1 + \hat{\beta}_5$ = 0.000296 - 0.003942 = 0.004238

ANCOVA Model

If the model also includes quantitative independent variable *(X_t)*, the model can be stated as:

Model with intercept dummy: $Y_t = \beta_0 + \beta_1 X_{1t} + \gamma_0 D_{1t} + \lambda_0 D_{5t} + u_t$

This model can be interpreted as:

Model for Tuesday, Wednesday and Thursday: $Y_t = \beta_0 + \beta_1 X_{1t} + u_t$
Model for Monday: $Y_t = (\beta_0 + \gamma_0) + \beta_1 X_{1t} + u_t$
Model for Friday: $Y_t = (\beta_0 + \lambda_0) + \beta_1 X_{1t} + u_t$

```
. reg y x1 d1 d5

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  3,   991) =   11.68
       Model |  .007808227     3  .002602742           Prob > F      =  0.0000
    Residual |  .220903311   991  .000222909           R-squared     =  0.0341
-------------+------------------------------           Adj R-squared =  0.0312
       Total |  .228711537   994  .000230092           Root MSE      =  .01493

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   1.90e-06   8.91e-07     2.13   0.033     1.50e-07    3.65e-06
          d1 |  -.0043131   .0012221    -3.53   0.000    -.0067113   -.0019149
          d5 |   .0039385   .0012221     3.22   0.001     .0015403    .0063368
       _cons |  -.0708331   .0333809    -2.12   0.034    -.1363383   -.0053278
------------------------------------------------------------------------------
```

Model with intercept and slope dummy variables:

$$Y_t = \beta_0 + \gamma_0 D_{1t} + \lambda_0 D_{5t} + \beta_1 X_{1t} + \gamma_1 D_{1t} X_{1t} + \lambda_1 D_{5t} X_{1t} + u_t$$

This model can be interpreted as:

Model for Tuesday, Wednesday and Thursday: $Y_t = \beta_0 + \beta_1 X_{1t} + u_t$
Model for Monday: $Y_t = (\beta_0 + \gamma_0) + (\beta_1 + \gamma_1) X_{1t} + u_t$
Model for Friday: $Y_t = (\beta_0 + \lambda_0) + (\beta_1 + \lambda_1) X_{1t} + u_t$

```
. g x1d1 = x1*d1

. g x1d5 = x1*d5

. reg y d1 d5 x1 x1d1 x1d5

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  5,   989) =    7.19
       Model |  .008022894     5  .001604579           Prob > F      =  0.0000
    Residual |  .220688644   989  .000223143           R-squared     =  0.0351
-------------+------------------------------           Adj R-squared =  0.0302
       Total |  .228711537   994  .000230092           Root MSE      =  .01494

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          d1 |  -.0541205    .086225    -0.63   0.530    -.2233254    .1150844
          d5 |   .0576593   .0862319     0.67   0.504    -.1115592    .2268777
          x1 |   1.92e-06   1.15e-06     1.67   0.096    -3.39e-07    4.18e-06
        x1d1 |   1.33e-06   2.30e-06     0.58   0.564    -3.19e-06    5.85e-06
        x1d5 |  -1.43e-06   2.30e-06    -0.62   0.533    -5.95e-06    3.08e-06
       _cons |  -.0716146   .0431142    -1.66   0.097    -.1562204    .0129911
------------------------------------------------------------------------------
```

Chow Test vs Dummy Variable Technique

If we would like to check whether the model should be separated into two sub-periods (2000-2001 and 2002-2004), the models can be stated as:

Model 2000-2001: $Y_t = \beta_0 + \beta_1 X_{1t} + u_{1t}$

Model 2002-2004: $Y_t = \gamma_0 + \gamma_1 X_{1t} + u_{2t}$

Chow Test: H_0: $\beta_0 = \gamma_0$ and $\beta_1 = \gamma_1$

Dummy Variable Regression Model

Model with intercept and slope dummy:

$$Y_t = \beta_0 + \delta_0 D_{YEARt} + \beta_1 X_{1t} + \delta_1 D_{YEARt} X_{1t} + u_t$$

where: D_{YEARt} = 1 for 2002-2004 and = 0 for 2000-2001.

The model can be interpreted as:

Model 2000-2001: $Y_t = \beta_0 + \beta_1 X_{1t} + u_{1t}$

Model 2002-2004: $Y_t = (\beta_0 + \delta_0) + (\beta_1 + \delta_1) X_{1t} + u_{2t}$

H_{01}: $\delta_0 = 0$ and $\delta_1 = 0$

```
. reg y x1

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  1,   993) =    4.47
       Model |  .001024966     1  .001024966           Prob > F      =  0.0347
    Residual |  .227686571   993  .000229292           R-squared     =  0.0045
-------------+------------------------------           Adj R-squared =  0.0035
       Total |  .228711537   994  .000230092           Root MSE      =  .01514

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   1.91e-06   9.04e-07     2.11   0.035     1.37e-07    3.69e-06
       _cons |  -.0713463    .033853    -2.11   0.035    -.1377779   -.0049147
------------------------------------------------------------------------------

. sca rss1=e(rss)

. sca n1=e(N)

. reg y x1 if dyear==0

      Source |       SS       df       MS              Number of obs =     400
-------------+------------------------------           F(  1,   398) =    4.43
       Model |  .001270974     1  .001270974           Prob > F      =  0.0359
    Residual |  .114079495   398  .000286632           R-squared     =  0.0110
-------------+------------------------------           Adj R-squared =  0.0085
       Total |   .11535047   399  .000289099           Root MSE      =  .01693

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   8.49e-06   4.03e-06     2.11   0.036     5.64e-07    .0000164
       _cons |  -.3139704   .1487967    -2.11   0.035    -.6064962   -.0214447
------------------------------------------------------------------------------

. sca rss2=e(rss)
```

```
. sca n2=e(N)

. reg y x1 if dyear==1

      Source |       SS          df       MS      Number of obs   =       595
-------------+----------------------------------  F(1, 593)       =      0.39
       Model |  .000074137         1  .000074137  Prob > F        =    0.5326
    Residual |  .112782707       593   .00019019  R-squared       =    0.0007
-------------+----------------------------------  Adj R-squared   =   -0.0010
       Total |  .112856844       594  .000189995  Root MSE        =    .01379

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   1.11e-06   1.78e-06     0.62   0.533    -2.39e-06    4.61e-06
       _cons |  -.0412568   .0673712    -0.61   0.541     -.173572    .0910584
------------------------------------------------------------------------------

. sca rss3=e(rss)

. sca n3=e(N)

. sca ChowTest=((rss1-rss2-rss3)/2)/((rss2+rss3)/(n2+n3-2*2))

. sca list ChowTest
  ChowTest =  1.8005421
```

Dummy Variable Technique

```
. g x1dyear = x1*dyear

. reg y dyear x1 x1dyear

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  3,   991) =    2.69
       Model |  .001849335     3  .000616445           Prob > F      =  0.0450
    Residual |  .226862202   991  .000228923           R-squared     =  0.0081
-------------+------------------------------           Adj R-squared =  0.0051
       Total |  .228711537   994  .000230092           Root MSE      =  .01513

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       dyear |   .2727136   .1521381     1.79   0.073    -.0258363    .5712635
          x1 |   8.49e-06   3.60e-06     2.36   0.019     1.42e-06    .0000156
     x1dyear |  -7.38e-06   4.10e-06    -1.80   0.072    -.0000154    6.66e-07
       _cons |  -.3139704   .1329766    -2.36   0.018    -.5749185   -.0530223
------------------------------------------------------------------------------

. test dyear x1dyear

 ( 1)  dyear = 0
 ( 2)  x1dyear = 0

       F(  2,   991) =    1.80
            Prob > F =    0.1657
```

Distinction between the two methods is that while Chow test is to test whether all estimators are the same or not, dummy variables technique is to separately test equality of each estimator for different sub-period -- some estimators may be different while some may not.

Interaction Effect

Model with interaction effect dummy:

$$Y_t = \beta_0 + \gamma_0 D_{1t} + \lambda_0 D_{JANt} + \delta_0 D_{1t} D_{JANt} + \beta_1 X_{1t} + u_t$$

where: D_{1t} = 1 for Monday and = 0 otherwise.
D_{JANt} = 1 for January and = 0 otherwise.

The meaning of $D_{1t}D_{JANt}$

$D_{1t}D_{JANt}$ = 1 for Monday in January and = 0 otherwise.

```
. g d1djan = d1*djan

. reg y d1 djan d1djan x1

      Source |       SS       df       MS              Number of obs =     995
-------------+------------------------------           F(  4,   990) =    7.55
       Model |  .006773371     4  .001693343           Prob > F      =  0.0000
    Residual |  .221938166   990   .00022418           R-squared     =  0.0296
-------------+------------------------------           Adj R-squared =  0.0257
       Total |  .228711537   994  .000230092           Root MSE      =  .01497

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          d1 |  -.0059289   .0012667    -4.68   0.000    -.0084147   -.0034432
        djan |   .0019612   .0017065     1.15   0.251    -.0013876    .0053099
      d1djan |    .004773   .0036262     1.32   0.188    -.0023429    .0118889
          x1 |   2.08e-06   8.98e-07     2.32   0.021     3.21e-07    3.85e-06
       _cons |  -.0769402    .033645    -2.29   0.022    -.1429639   -.0109166
------------------------------------------------------------------------------
```

Proof – Oneway ANOVA & ANOVA Model

```
. oneway y day, tab

            |           Summary of Y
        day |        Mean   Std. Dev.       Freq.
------------+------------------------------------
          1 |  -.00402112   .01629203         199
          2 |   .00044255   .01373011         199
          3 |   .00041396   .01598835         199
          4 |   .00003082    .0147649         199
          5 |   .00423812   .01389622         199
------------+------------------------------------
      Total |   .00022087   .01516879         995

                        Analysis of Variance
    Source              SS         df      MS            F     Prob > F
------------------------------------------------------------------------
Between groups      .00681682      4   .001704205      7.60     0.0000
 Within groups     .221894717    990   .000224136
------------------------------------------------------------------------
    Total          .228711537    994   .000230092

Bartlett's test for equal variances:  chi2(4) =   9.7365  Prob>chi2 = 0.045
```

```
. reg y d2 d3 d4 d5

      Source |       SS           df       MS      Number of obs   =       995
-------------+----------------------------------   F(4, 990)       =      7.60
       Model |  .00681682          4  .001704205   Prob > F        =    0.0000
    Residual | .221894717        990  .000224136   R-squared       =    0.0298
-------------+----------------------------------   Adj R-squared   =    0.0259
       Total | .228711537        994  .000230092   Root MSE        =    .01497

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          d2 |   .0044637   .0015009     2.97   0.003     .0015184    .0074089
          d3 |   .0044351   .0015009     2.96   0.003     .0014898    .0073803
          d4 |   .0040519   .0015009     2.70   0.007     .0011067    .0069972
          d5 |   .0082592   .0015009     5.50   0.000      .005314    .0112045
       _cons |  -.0040211   .0010613    -3.79   0.000    -.0061037   -.0019385
------------------------------------------------------------------------------
```