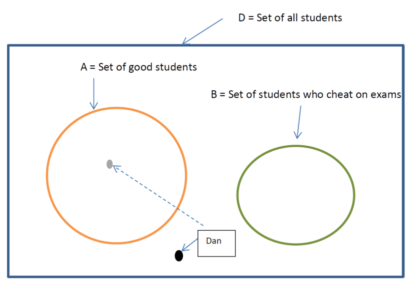
Solution: Quiz 2

1. Let the domain D be the set of students. Suppose Dan $\in D$.
Use the **diagram** to show that the following argument is valid or invalid.
"None of the good students cheats on the exam."
"Dan does not cheat on the exam."
$\therefore$ "Dan is a good student."

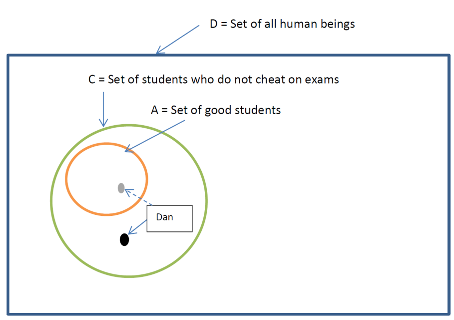
Solution: To use the diagram, let A be the set of good students, and
let B be the set of students who cheat on exams.
Then
- the first premise "None of the good students cheats on the exam." implies "**All good students do not cheat on the exams**" (i.e. being good students implies not cheating on the exam), or "$A \subseteq B^c$." or "$A \cap B = \emptyset$" where B^c is the compliment of set B.
- the second premise "Dan does not cheat on the exams." implies that Dan $\notin B$,
- the conclusion implies that "Dan is in the set A."

From the diagram, since we only know that "Dan" is not an element in B , it is possible that "Dan" is either an element in A or not an element in A. That is, the conclusion could be false. Therefore the argument is **invalid**.



Remarks: The following is an alternative solution. let A be the set of good students, and
let C be the set of students who **do not** cheat on exams.
Then, (i) the first premise implies "$A \subseteq C$." (ii) the second premise implies Dan $\in C$, (iii) the conclusion implies "Dan is in the set C."

From the diagram, since we only know that "Dan" is in C , it is possible that "Dan" is either an element in A or not an element in A. That is, the conclusion could be false and the argument is **invalid**.



2. Let $A = \{-1, 1\}$ and $B = \{1, 2, 3\}$. Let $D = \{(1,1), (0,-1)\}$ be the domain of variable (x, y) for the predicate $P(x, y)$ which is defined as

$$P(x,y) : \sim [\exists a \in A,\ \forall b \in B, \quad ab \geq x + y] .$$

Determine the truth set of the predicate $P(x, y)$.

Solution: First, we can simplify $P(x, y)$ by finding the negation of $[\exists a \in A,\ \forall b \in B, \quad ab \geq x + y]$:

$$P(x,y) : \sim [\exists a \in A,\ \forall b \in B, \quad ab \geq x + y] \ = \ [\forall a \in A,\ \exists b \in B, \quad ab < x + y] .$$

To find the truth set T_p of $P(x, y)$, we have to consider all elements in D that make $P(x, y)$ true.

(i) For $(x, y) = (1, 1) \in D$, we have $x + y = 2$ and $P(1, 1) : [\forall a \in A,\ \exists b \in B, \quad ab < 2]$ is **true** because:
- for $a = -1 \in A$, there exists $b \in B$ (e.g. $b = 1$) such that $ab = -1 < 2$ is true (we can also use $b = 2$ or $b = 3$);
- for $a = 1 \in A$, there exists $b \in B$ (e.g. $b = 1$) such that $ab = 1 < 2$ is true.

$\therefore (1, 1) \in T_p$

(ii) For $(x, y) = (0, -1) \in D$, we have $x + y = -1$ and $P(0, -1) : [\forall a \in A,\ \exists b \in B, \quad ab < -1]$ is **false** because when we can find $\boxed{a = 1 \in A}$ such that all values of $b \in B = \{1, 2, 3\}$ make $ab < -1$ false:
- for $b = 1$, $ab = 1 < -1$ is false;
- for $b = 2$, $ab = 2 < -1$ is false;
- for $b = 3$, $ab = 3 < -1$ is false.

$\therefore (0, -1) \notin T_p$

Hence $T_p = \{(x, y) \in D : \ P(x, y) \text{ is true.}\} = \{(1, 1)\}$. ∎