

Chapter 12

Solution to Problems

11) For Question (a) and (b), you can solve them by setting $MC = MR$.

(a)

Using the monopoly mid-point rule, $P^* = (70 + 10) / 2 = 40$

Using the demand curve to solve for Q^* , $40 = 70 - 0.5Q$ so $Q^* = 60$.

(b)

Softco sold the first 60 units at the price of 40

The residual demand that is not fulfilled by softco is the horizontal difference between the market demand and the first 60 units sold.

From $P = 70 - 0.5Q$ the market demand is $Q_{mkt} = 140 - 2P$.

The residual demand is $Q_{res} = Q_{mkt} - 60 = 80 - 2P$.

Rearrange Q_{res} into the inverse demand, we get $P = 40 - 0.5Q$.

Next, Softco maximizes profit subject to the residual demand.

(Here, you can also solve by setting $MR = MC$.)

Using the monopoly mid-point rule, $P_2^* = (40 + 10) / 2 = 25$.

Using the residual demand curve to solve for

Q_2^* , $25 = 40 - 0.5Q$ so $Q_2^* = 30$.

In conclusion, Softco sells another 30 units at $P = 25$. The total output sold = $60 + 30 = 90$.

(c)

Please see another solution note for this question.

- 12.) if there were no subscription charge, each consumer would realize a consumer surplus of $0.5 \cdot (10-2)^2 = 32$. This means that each consumer will be willing to buy electricity as long as the subscription charge is less than 32.

With 100 consumers, the electric utility can then charge each customer a subscription fee of \$12 to cover its fixed costs of \$1200, leaving each consumer with a consumer surplus of $32 - 12 = 20$. So the total revenue from the usage charge $100(8) = 800$. Total revenue will just cover total cost, and the firm will earn zero economic profit.

- 14.) a) With third-degree price discrimination the firm should set $MR = MC$ in each market to determine price and quantity. Thus, in Europe setting $MR = MC$

$$70 - 2Q_E = 10$$
$$Q_E = 30$$

At this quantity, price will be $P_E = 40$. Profit in Europe is then $\pi_E = (P_E - 10)Q_E = (40 - 10)30 = 900$. Setting $MR = MC$ in the US implies

$$110 - 2Q_U = 10$$
$$Q_U = 50$$

At this quantity price will be $P_U = 60$. Profit in the US will then be $\pi_U = (P_U - 10)Q_U = (60 - 10)50 = 2500$. Total profit will be $\pi = 3400$.

b) If the firm can only sell the drug at one price, it will set the price to profit. The total demand the firm will face is $Q = Q_E + Q_U$. In this case

The inverse demand is then $P = 90 - 0.5Q$. Since $MC = 10$, setting $MR = MC$ implies

$$90 - Q = 10$$

$$Q = 80$$

At this quantity price will be $P = 50$. If the firm sets price at 50, the firm will sell $Q_E = 20$ and $Q_U = 60$. Profit will be $\pi = 50(80) - 10(80) = 3200$

20.) $Q_1 = 750 - 4P_1 \rightarrow P_1 = 187.5 - (1/4)Q_1$. This implies

$$MR_1 = 187.5 - (1/2)Q_1.$$

$Q_2 = 850 - 2P_2 \rightarrow P_2 = 425 - (1/2)Q_2$. This implies $MR_2 = 425 - Q_2$.

When we have limited capacity we solve the following two equations: $MR_1 = MR_2 \rightarrow 187.5 - (1/2)Q_1 = 425 - Q_2$
(Equate the MRs) $Q_1 + Q_2 = 500$ (Quantities sold must add up to capacity)

We thus have two equations in two unknowns.

$$187.5 - (1/2)Q_1 = 425 - Q_2$$

$$Q_1 + Q_2 = 500$$

Solving these equations yields:

$$Q_1 = 175.$$

$$Q_2 = 325.$$

Plugging these back into the inverse demand curves gives us the profit-maximizing prices:

$$P_1 = 187.5 - (1/4)(175) = 143.75.$$

$$P_2 = 425 - (1/2)(325) = 262.5.$$

- 22.) a) Without bundling the best the firm can do is set the price of airfare at \$800 and the price of hotel at \$800. In each case the firm attracts a single customer and earns profit of \$500 from each for a total profit of \$1000. The firm could attract two customer for a total of \$800 profit, less profit than the \$800 price.
- b) With bundling, the best the firm can do is charge a price of \$900. The firm could raise its price to \$1000, but then it would only attract one customer and total profit would be \$400. Notice that with bundling the firm cannot do as well as it could with mixed bundling.
- c) Because customer 1 has a willingness-to-pay for airfare below marginal cost and customer 3 has willingness-pay for hotel below marginal cost, the firm can potentially earn greater profits through mixed bundling. In this problem, if the firm charges \$800 for airfare only, \$800 for hotel only, and \$ will purchase the bundle, then customer 1 will purchase hotel only, customer 2 will purchase the bundle, and customer 3 will purchase airfare only. This will earn the firm \$1400 profit, implying that mixed bundling is the best option in this problem.

c) Now, reconsider your answer to (b). Find the structure of prices and quantities in each of the two blocks that maximizes profit. In other words, you no longer assume that the price and quantity that you determined in (a) is fixed. Instead, you must find the optimal price for both blocks.

$$Q_2 = \frac{Q_1 + 120}{2} \quad (\text{mid point})$$

$$PS = TR - VC$$

$$PS = (Q_1 \cdot P_1) + (P_2)(Q_2 - Q_1) - 10Q_2$$

Find Q_1^* , Q_2^* , P_1 , P_2

$$\pi = (Q_1) \left(70 - \frac{Q_1}{2}\right) + \left(70 - \frac{Q_1}{2}\right) (Q_2 - Q_1) - 10Q_2$$

$$\pi = \cancel{70Q_1} - \frac{Q_1^2}{2} + 70Q_2 - \cancel{70Q_1} - \frac{Q_1^2}{2} + \frac{Q_1Q_2}{2} - 10Q_2$$

$$\frac{\partial \pi}{\partial Q_1} = -Q_1 + \frac{1}{2}Q_2$$

$$\frac{\partial \pi}{\partial Q_1} = 0 \rightarrow 2Q_1 = Q_2$$

$$\frac{\partial \pi}{\partial Q_2} = 60 - Q_2 + \frac{1}{2}Q_1$$

$$\frac{\partial \pi}{\partial Q_2} = 0 \rightarrow Q_2 = \frac{1}{2}Q_1 + 60$$

Solving

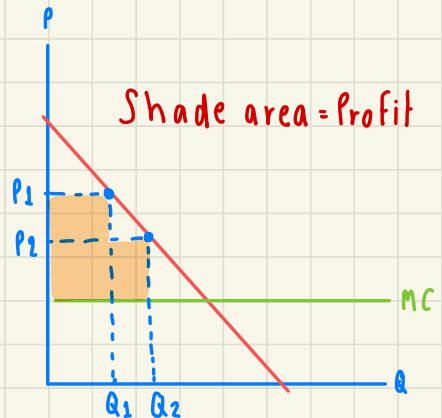
$$2Q_1 = \frac{1}{2}Q_1 + 60$$

$$Q_1 = 40$$

$$Q_2 = 2Q_1 = 80$$

$$P_1 = 50$$

$$P_2 = 30$$



12.** Consider a bar whose owner plans to set profit-maximizing two-part tariff (entry fee and per-drink price) on two types of customers. The owner would like to welcome both types into his bar, meaning that he will not charge an entry fee that is too high.

There are 20 people of the X-type whose individual demand is given by $P = 10 - Q_x$. There are 30 people of the Y-type whose individual demand is given by $P = 10 - 2Q_y$. The $MC = AC = \$2$ per drink.

Find the optimal entry fee and per-drink price. Also, calculate the profit the bar can make from these 50 customers.

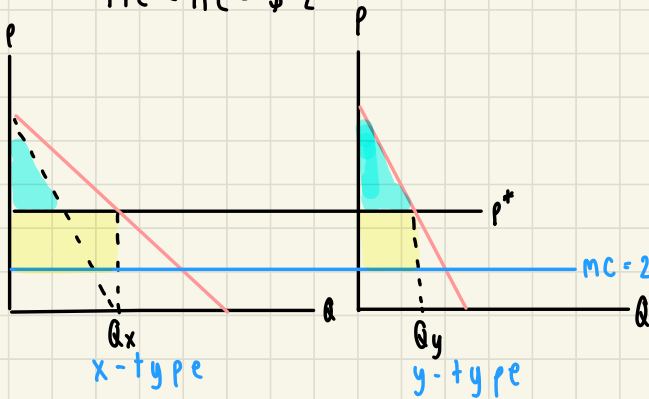
x-type

$$P = 10 - Q_x \quad 20 \text{ people}$$

y-type

$$P = 10 - 2Q_y \quad 30 \text{ people}$$

$$MC = AC = \$2$$



$$\text{total profit} = TR - TC$$

$$\text{total Revenue} = 50 \left(\frac{1}{2} (10 - P) Q_y \right) + 20 Q_x P^* + 30 Q_y P^*$$

$$= 50 \left(25 - 5P + \frac{P^2}{4} \right) + 200P - 20P^2 + 150P - 15P^2$$

$$= 12.5P^2 - 250P + 1250 + 200P - 20P^2 + 150P - 15P^2$$

$$Q_y = \frac{10 - P}{2}$$

$$Q_x = 10 - P$$

total cost

$$\begin{aligned} &= 20 (2 \cdot Q_x) + 30 (2 \cdot Q_y) \\ &= 400 (10 - P) + 600 \cdot \left(\frac{10 - P}{2} \right) \\ &= 400 - 40P + 300 - 30P \\ &= 700 - 70P \end{aligned}$$

total profit

$$\begin{aligned} &= 12.5P^2 - 250P + 1250 + \\ &\quad 200P - 20P^2 + 150P - 15P^2 - 700 \\ &\quad + 70P \end{aligned}$$

$$\frac{d\pi}{dP} = 25P - 250 + 200 - 40P + 150 - 30P + 70 + 30 = 0$$

$$P^* = 3.78 \quad \text{- per drink price}$$

Optional entry fee

$$= (10 - 3.78) \left(\frac{10 - 3.78}{2} \right) \frac{1}{2}$$

$$= 9.67 \$ *$$

$$\pi = (50) 9.675 + 1.78 (3.11) (30) + 1.78 (6.22) (20)$$

$$= 483.75 + 166.074 + 221.432$$

$$= 871.256 *$$