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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i) = 5a + b\sum_{i=1}^{5} x_i$

   $= 5a + b(x_1 + x_2$

   b. $\sum_{y=0}^{5} f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

   c. $\sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2(10)+1)}{6} = 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y) = \sum_{x=1}^{2}[2x+2+3] = \sum_{x=1}^{2} 2x+5 = 2(1+2(2)+5 = 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b

   $f(x) = P(X=x) : 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$

   $8b = 1$

   $b = 1/8$

   b. Find the answer for $P(X \le 2)$

   $P(X \le 2) = 1 - P(X > 2)$

   $= 1 - P(X=3) - P(X=4)$

   $= 1 - (0.5)(1.25) - 0.25(0.125) = 0.90625$

   c. Find the answer for $P(-2 \le X \le 3)$

   $P(-2 \le X \le 3) = 1 - P(X=4)$

   $= 1 - (0.25)(0.125)$

   $= 0.96875$

   d. Find the answer for $P(X \ge 1)$

   $P(X \ge 1) = 1 - P(X > 1)$

   $= 1 - P(X=0) - P(X=-1) - P(X=-2)$

   $= 1 - (2.25)(0.125) - (1)(0.125) - 0.5(0.125)$

   $= 0.53125$

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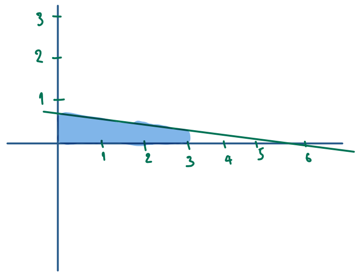
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is $(x,y)=(6,\frac{2}{3})$

$$f(x)=-\frac{1}{9}x+\frac{6}{9}, 0\le x\le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1\le X\le 3)$

$$P(1\le x<3)=\int_1^3 f(x)\,dx$$

$$=-\frac{1}{18}x^2+\frac{6}{9}x\Big|_1^3$$

$$=\left[\frac{-(3)^2}{18}+\frac{6(3)}{9}\right]-\left[\frac{-(1)^2}{18}+\frac{6(1)}{9}\right]=\frac{-9}{18}+\frac{18}{9}+\frac{1}{18}-\frac{6}{9}=\frac{-8}{18}+\frac{12}{9}=\frac{16}{18}$$

c. Find the answer for $P(X\ge 2)$

$$P(x\ge 2)=\int_2^3 f(x)\,dx$$

$$=-\frac{1}{18}x^2+\frac{6}{9}x\Big|_2^3$$

$$=\left[\frac{-(3)^2}{18}+\frac{6(3)}{9}\right]-\left[\frac{-(2)^2}{18}+\frac{6(2)}{9}\right]=\frac{7}{18}$$

d. Find the expected value of X

$$E(x)=\int_{-\infty}^{\infty} x f(x)\,dx$$

$$=\int_0^3 x\left(-\frac{1}{9}x+\frac{6}{9}\right)dx$$

$$=\int_0^3 -\frac{1}{9}x^2+\frac{6}{9}x\,dx$$

$$=\frac{-x^3}{27}+\frac{6x^2}{18}\Big|_0^3$$

$$=\frac{-(3)^3}{27}+\frac{6(3)^2}{18}$$

$$=-\frac{27}{27}+\frac{6\cdot 9}{18}$$

$$=-1+3$$

$$=2$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

x/y	1	2	3	4	5	6	
0	1/12	1/12	1/12	1/12	1/12	1/12	1/2
1	1/12	1/12	1/12	1/12	1/12	1/12	1/2
	1/6	1/6	1/6	1/6	1/6	1/6	1

   b. Find the marginal probability distribution function (PDF) of X

The marginal prob of x $p(x=x)$ is represented ingreen

   c. Find the marginal probability distribution function (PDF) of Y

The marginal prob of y $p(y=y)$ is represented in pink

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

x	P(x=x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

   e. Find the expected value of X given Y is equal to 1

$$E(x|y=1) = \sum x_i P(x=x_i|y=1) = \frac{\sum x_i P(x=x_i, y=1)}{P(y=1)} = \frac{1}{P(y=1)} \sum x_i P(x=x_i, y=1)$$

$$= \frac{1}{0.5}\left[(1\cdot\tfrac{1}{12})+(2\cdot\tfrac{1}{12})+(3\cdot\tfrac{1}{12})+(4\cdot\tfrac{1}{12})+(5\cdot\tfrac{1}{12})+(6\cdot\tfrac{1}{12})\right] = \frac{7}{2}$$

   f. Find the variance of X given Y is equal to 1

$$V(x|y=1) = \sum (x - E(x|y=1))^2 \cdot P(x|y=1)$$

$$= \left(\frac{25}{4}\cdot\frac{1}{6}\right) + \left(\frac{9}{4}\quad\frac{1}{6}\right) + \left(\frac{1}{4}\cdot\frac{1}{6}\right) + \left(\frac{1}{4}\cdot\frac{1}{6}\right) + \left(\frac{9}{4}\cdot\frac{1}{6}\right) + \left(\frac{25}{4}\cdot\frac{1}{6}\right)$$

$$= \frac{120}{24} = \frac{10}{3}$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$

$= \frac{1}{N} E\,(x_1 + x_2 + x_3)$

$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3)\right]$

$= \frac{1}{3}\left[\mu_x + \mu_x + \mu_x\right]$

$= \frac{1}{3} \cdot 3\mu_x = \mu_x = \bar{x}$

$Var(\bar{x}) = Var\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$

$= \frac{1}{N^2} Var(x_1 + x_2 + x_3)$

$= \frac{1}{N^2}\left[Var(x_1) + Var(x_2) + Var(x_3)\right]$

$= \frac{1}{N^2}$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$

$= \frac{1}{N} E(x_1 + x_2 + x_3 + x_4)$

$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3) + E(x_4)\right]$

$= \frac{1}{4}\left[\mu_x + \mu_x + \mu_x + \mu_x\right]$

$= \frac{1}{4} \cdot 4\mu_x = \mu_x$

$Var(\bar{x}) = Var\left(\frac{1}{N}\sum_{x=1}^{4} x_i\right)$

$= \frac{1}{N^2} Var(x_1 + x_2 + x_3 + x_4)$

$= \frac{1}{N^2}\left[Var(x_1) + Var(x_2) + Var(x_3) + Var(x_4)\right]$

$= \frac{1}{4^2} \cdot 4\sigma_x^2 = \frac{\sigma_x^2}{4} = 0.25\,\sigma_x^2$

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$\tilde{X} = \frac{1}{4}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$E(\tilde{X}) = E\left(\frac{1}{N}\sum_{i=1}^{4} X_i\right)$$

$$= \frac{1}{4}E(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4}[0.5E(X_1) + E(X_2) + 0.5E(X_3) + 2E(X_4)]$$

$$= \frac{1}{4}(0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$$

$$= \frac{1}{4} \cdot 4\mu_x = \mu_x$$

$\tilde{X}$ is unbiased estimator of μ

$$Var(\tilde{X}) = Var\left(\frac{1}{N}\sum_{i=1}^{4} X_i\right)$$

$$= \frac{1}{4^2}Var(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4^2}[Var(0.5X_1) + Var(X_2) + Var(0.5X_3) + Var(2X_4)]$$

$$= \frac{1}{4^2}[0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2]$$

$$= \frac{5.5\sigma_x^2}{16}$$

$$= 0.34\sigma_x^2$$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$\bar{x}$ is more efficient estimator of μ_x than $\tilde{x}$ because it has smaller varience.