Topic 1 Part 2

Consumer Preferences and the Concept of Utility (Chapter 3)

Summary

THREE Assumptions on **Preference**

- Completeness
  - a consumer can rank any two bundles
- Transitivity
  - if A is preferred to B and B is preferred to C, then A is preferred to C
- Monotonicity
  - more is better

Summary

TWO Assumptions on **Utility Function**

- Monotonicity (more gives higher utility)
  - MU of each good is positive
  - U’ is positive
- Diminishing MU
  - MU of each good is decreasing in the good
  - U’’ is negative

Indifference Curves

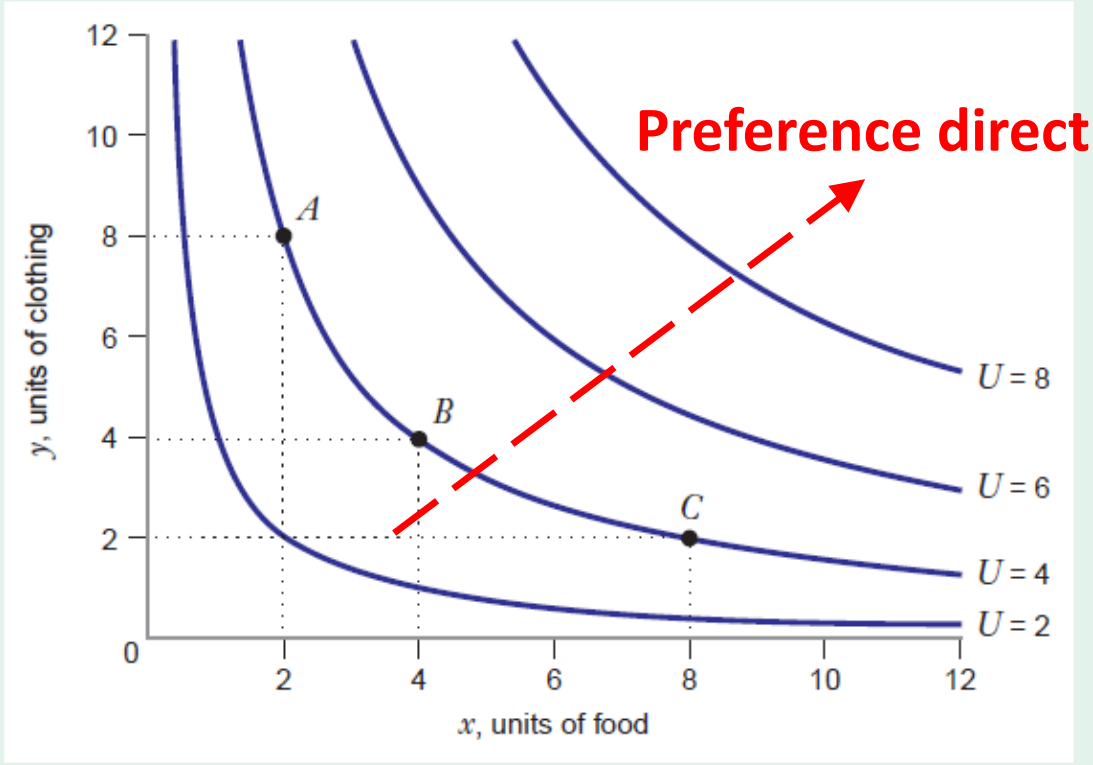
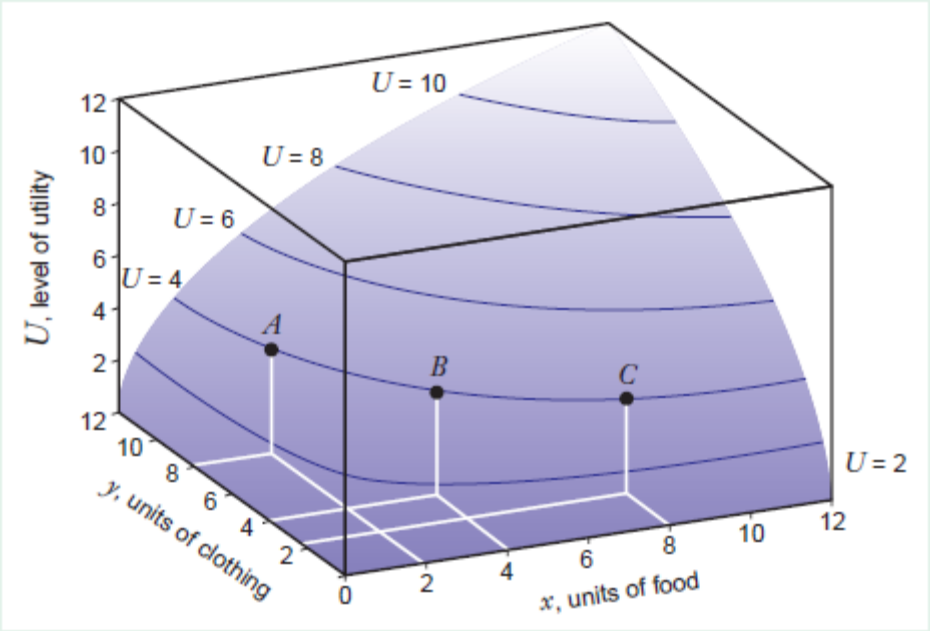
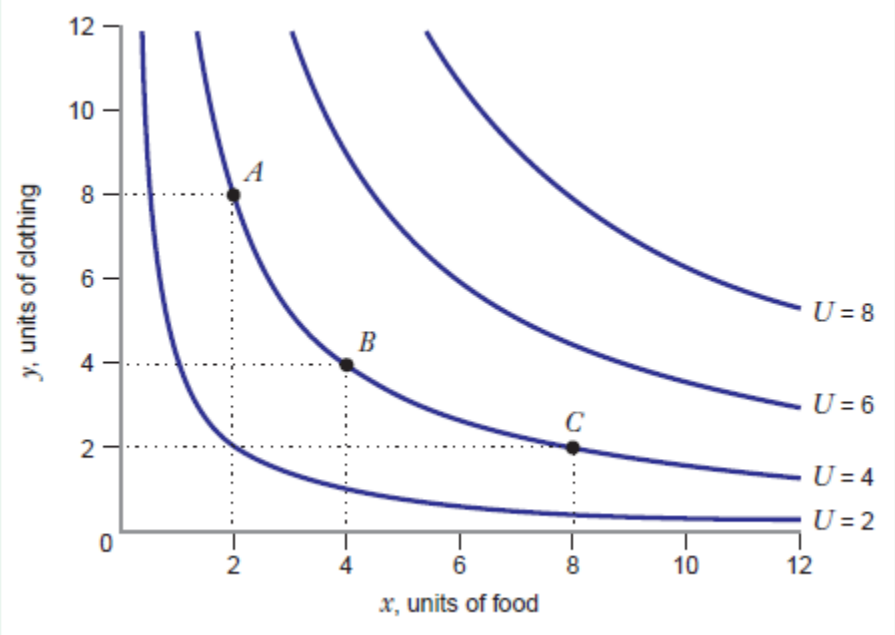


FIGURE 3.5 Indifference Curves for the Utility Function $U = \sqrt{xy}$
The utility is the same for all baskets on a given indifference curve. For example, the consumer is indifferent between baskets *A*, *B*, and *C* in the graph because they all yield the same level of utility ($U = 4$).


U = 10
U = 8
U = 6
U = 4
U = 2
A
B
C
U, level of utility
y, units of clothing
x, units of food
0
2
4
6
8
10
12


A
B
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y, units of clothing
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0
2
4
6
8
10
12

Indifference Curves

Example

$$U = xy^2$$

Check that underlying preferences are complete, transitive, and monotonic.

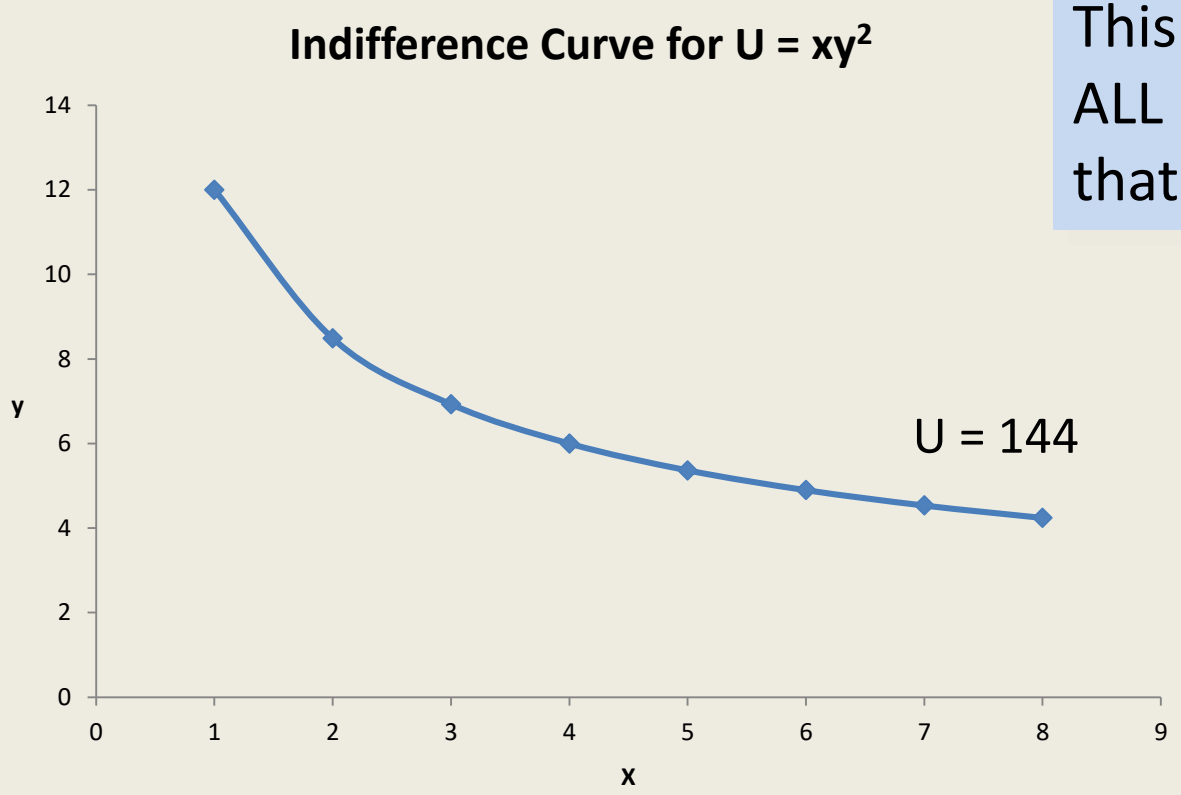
$$MU_x = y^2$$

$$MU_y = 2xy$$

$$for \quad U = 144$$

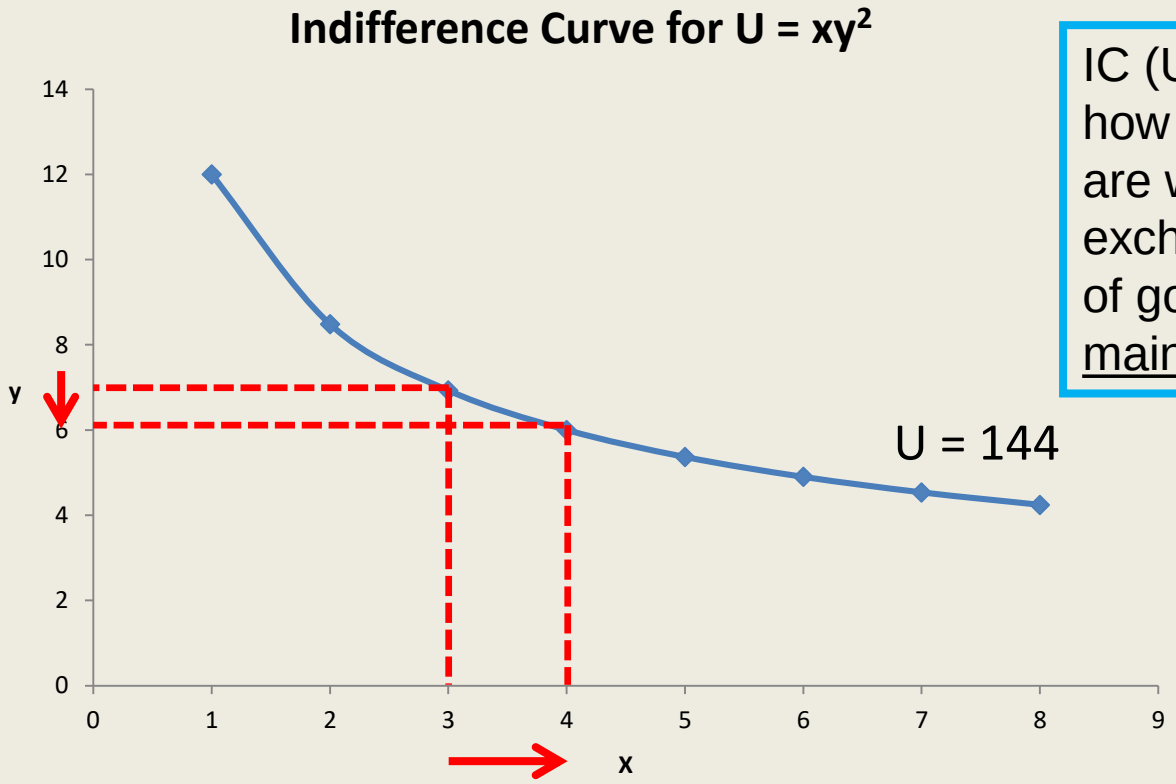
x	y	xy^2
8	4.24	143.8
4	6	144
3	6.93	144.07
1	12	144

Indifference Curves



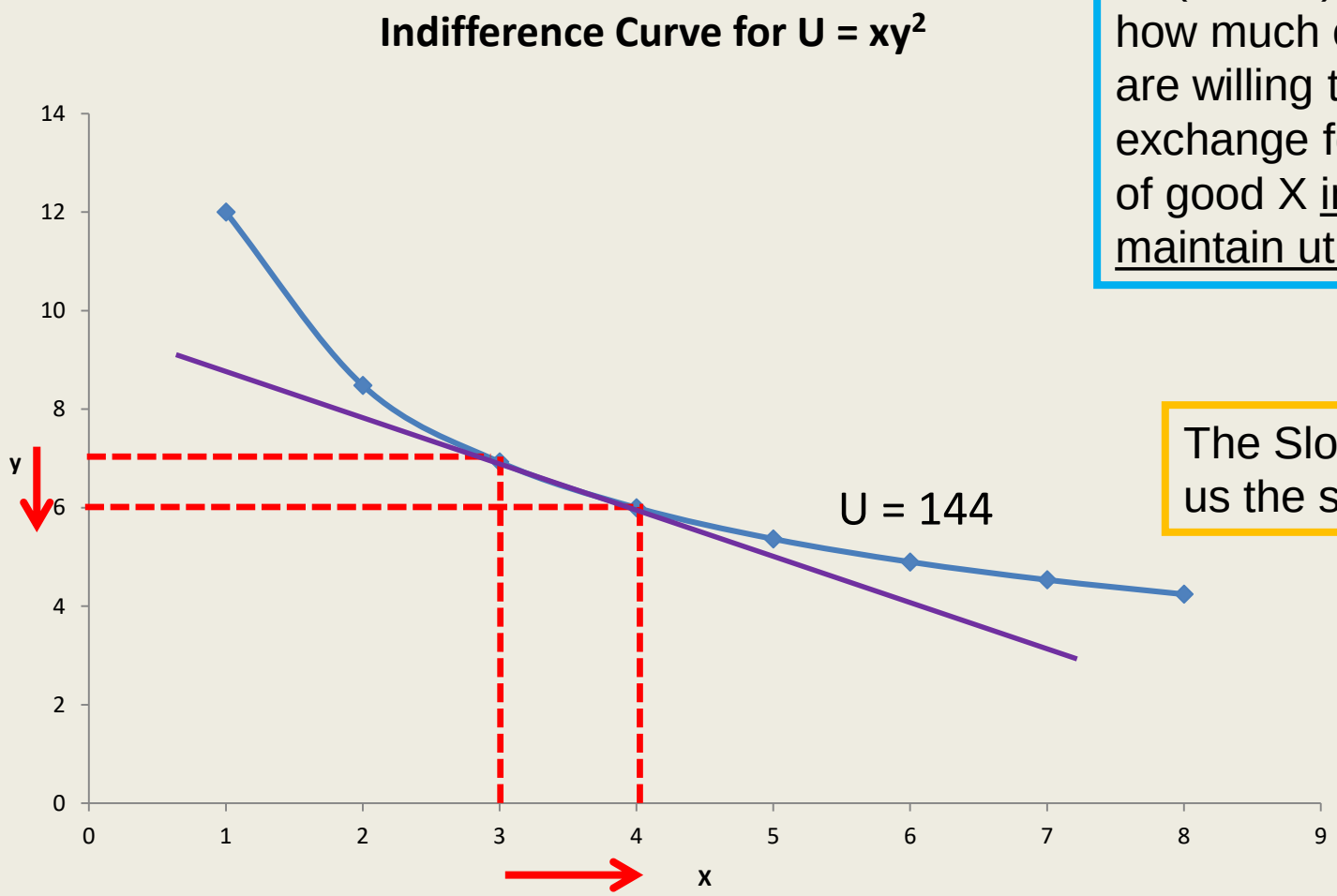
This IC tells us about ALL baskets of goods that give U = 144.

Indifference Curves



IC (U=144) tells us about how much of good Y we are willing to sacrifice in exchange for one more unit of good X <u>in order to maintain utility level of 144</u>

Indifference Curves



IC (U=144) tells us about how much of good Y we are willing to sacrifice in exchange for one more unit of good X in order to maintain utility level of 144

The Slope of IC tells us the same thing.

Marginal Rate of Substitution

The slope of IC is the "negative" Marginal Rate of Substitution (MRS).

- Formally speaking, it is the rate at which the consumer will give up one good to get more of another, while maintaining the same level of utility.
- Mathematically speaking, the slope of any curve, including IC, tells us the same thing: **how Y will change when X increases by one unit**.
- Simply speaking, it tell us how much Y the consumer is willing to give up in exchange for one more unit of X.
- For example, when –MRS = –5 (remember IC has negative slope), this means that the consumer is willing to give up 5 units of Y (i.e. **Y decreases by 5 units**) for **one more unit of X**.

Marginal Rate of Substitution

The slope of IC is the **derivative of the equation that describes IC**.

For example, for utility function $U = XY^2$, we can write the equation for IC where U = 144 as follows:

$144 = XY^2$ $\quad Y^2 = 144/X$ $\quad Y = 12/\sqrt{X}$ $\quad Y = 12X^{(-1/2)}$

Slope of IC = $dY/dX = -6X^{(-3/2)}$

Alternatively, $\mathbf{MRS_{XY} = MU_X/MU_Y}$.

Note that $MU_X = Y^2$ and $MU_Y = 2XY$, so $MRS_{XY} = Y/2X$.

Recall that $Y = 12X^{(-1/2)}$, so $MRS_{XY} = 6X^{(-3/2)}$

Beware of negative sign!

Marginal Rate of Substitution

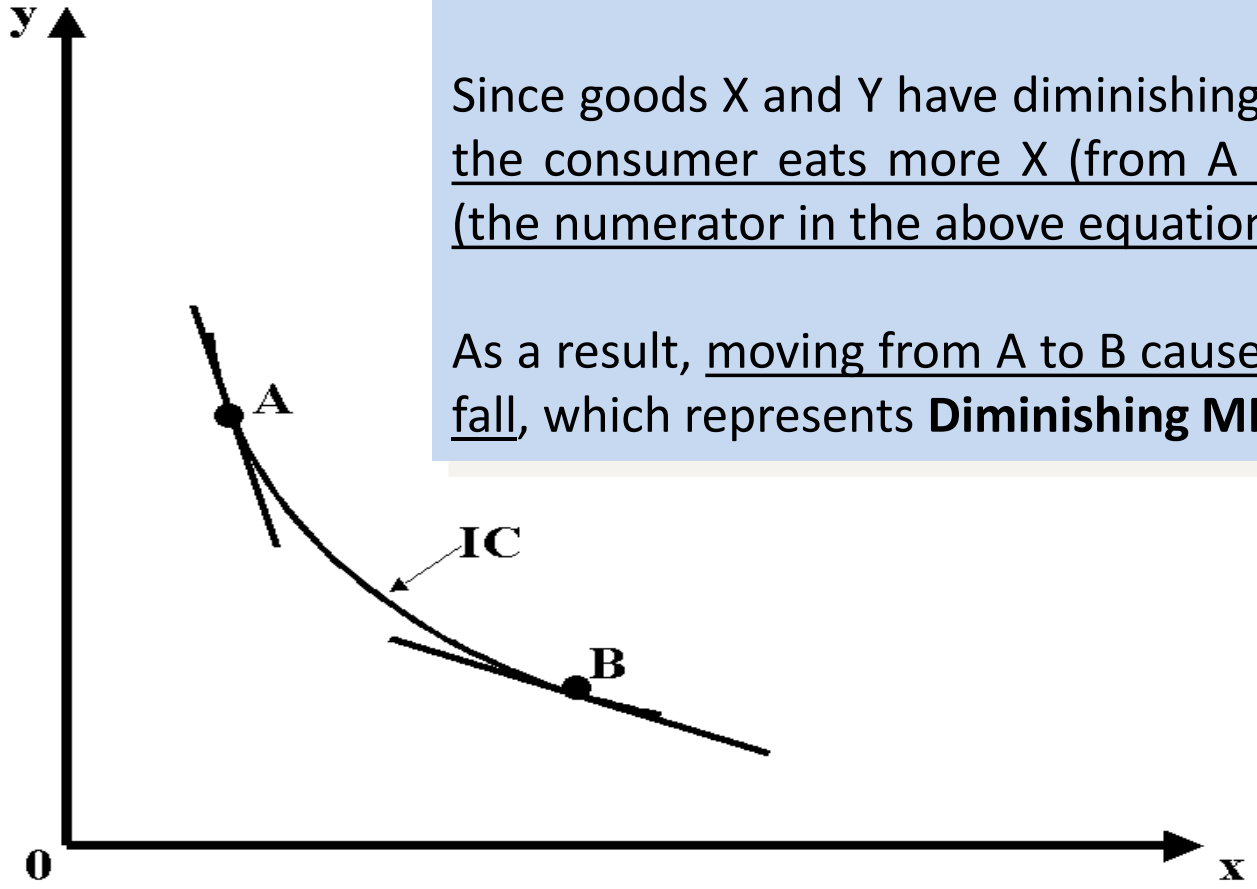
Observe that $-MRS_{XY} = dY/dX = -6X^{(-3/2)}$

Hence, the slope of IC is given by

1) $-MRS_{XY}$ OR 2) dY/dX

Proof

Marginal Rate of Substitution

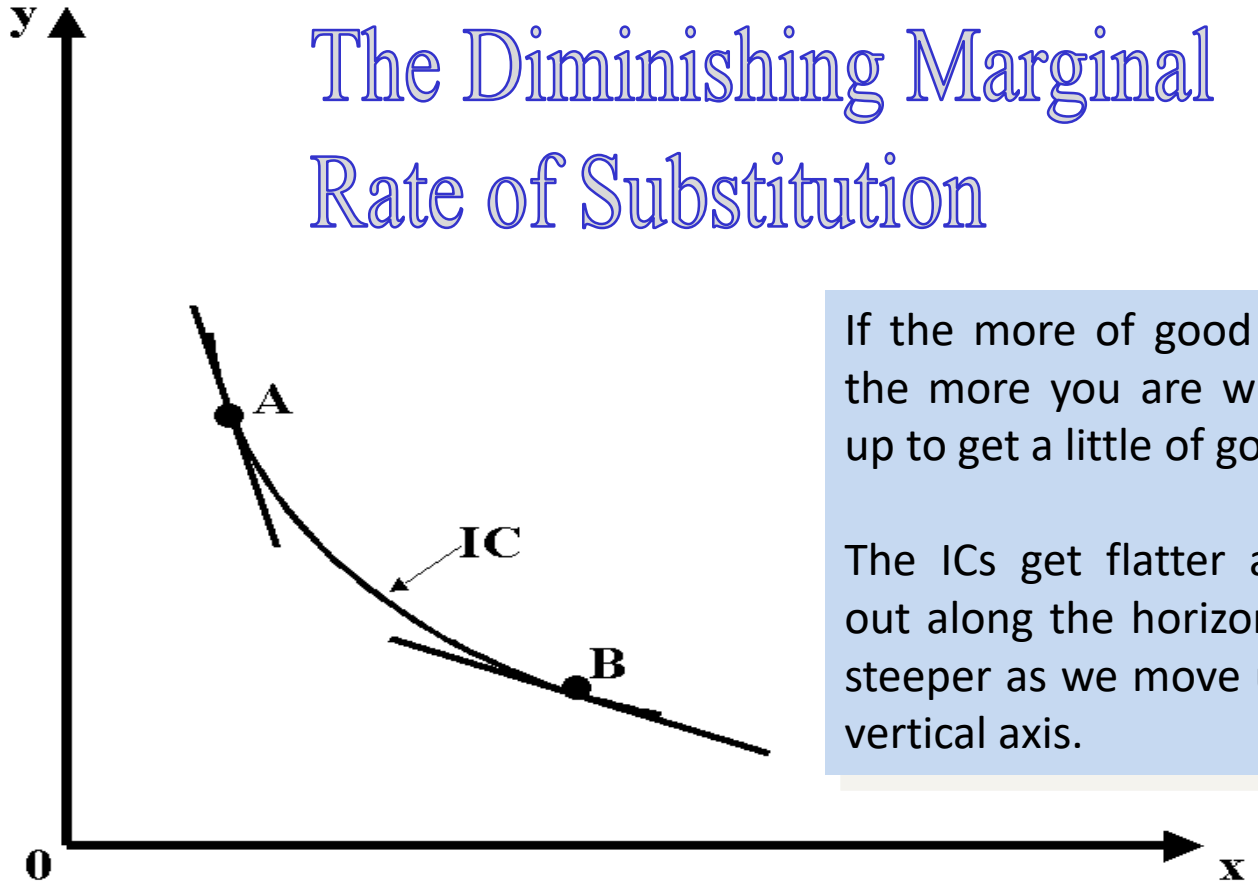


Note that MRSxy = MUx/MUy.

Since goods X and Y have diminishing MU, when the consumer eats more X (from A to B), MUx (the numerator in the above equation) falls.

As a result, moving from A to B causes MRSxy to fall, which represents **Diminishing MRSxy**.

Marginal Rate of Substitution



The Diminishing Marginal Rate of Substitution

If the more of good x you have, the more you are willing to give up to get a little of good y.

The ICs get flatter as we move out along the horizontal axis and steeper as we move up along the vertical axis.

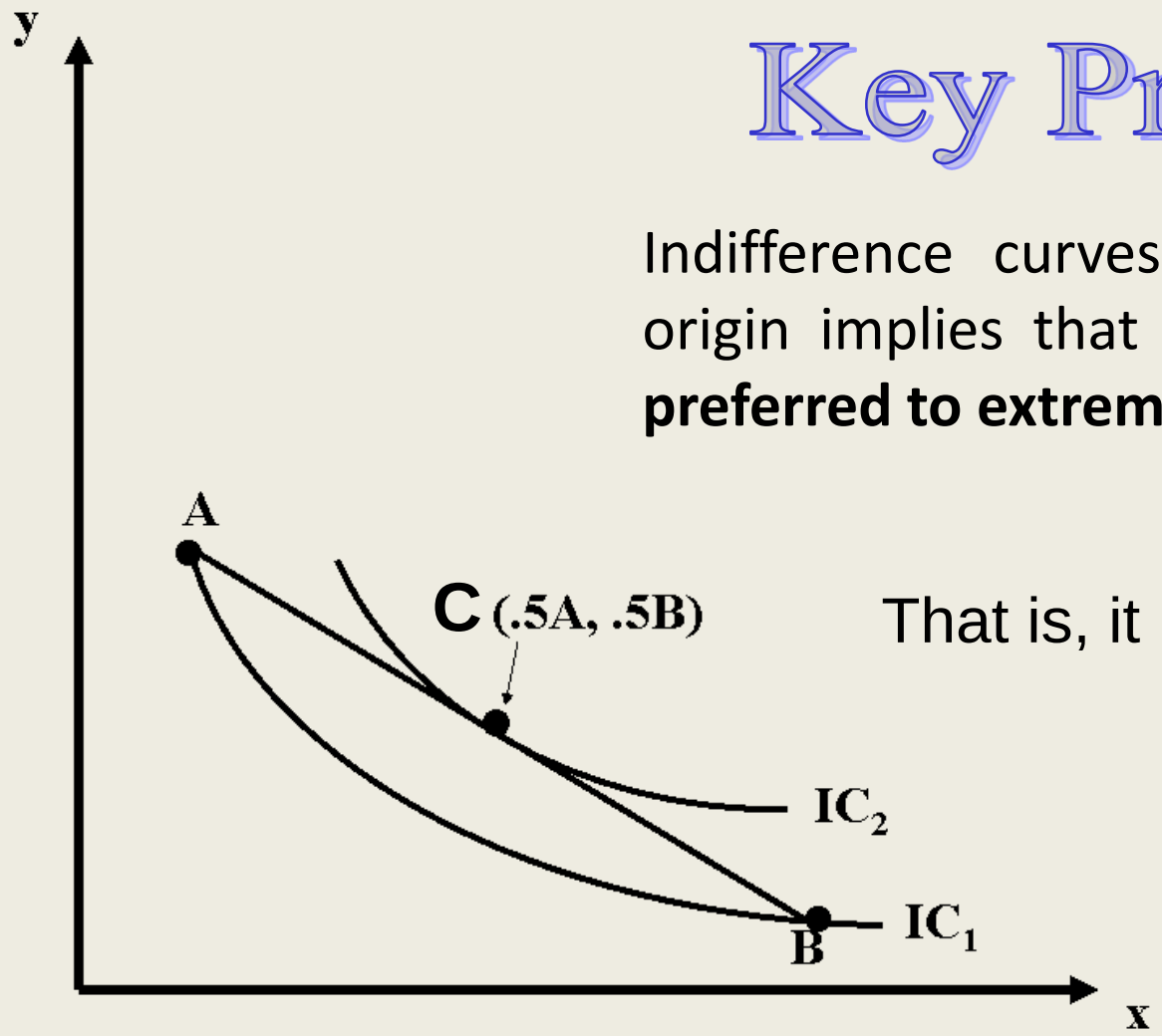
Marginal Rate of Substitution

Key Points about the shape of IC $\textbf{IC Slope} = -MRS_{XY} = dY/dX$

- Monotonicity implies positive MU. When MUx and MUy are positive, MRS is positive, which implies that **IC has a negative slope**.

- When MRS is diminishing (due to diminishing MUx and MUy), **ICs are convex to the origin**. This implies that average bundles are preferred to extreme bundles.

Indifference Curves



Key Property

Indifference curves are convex to the origin implies that **average bundle C is preferred to extreme bundles A and B.**

That is, it lies on a higher IC.

LEARNING-BY-DOING EXERCISE 3.3

Indifference Curves with Diminishing $MRS_{x,y}$

Suppose a consumer has preferences between two goods that can be represented by the utility function $U = xy$. For this utility function, $MU_x = y$ and $MU_y = x$.[6]

Problem

(a) On a graph, draw the indifference curve associated with the utility level $U_1 = 128$. Then answer the following questions:

1. Does the indifference curve intersect either axis?
2. Does the shape of the indifference curve indicate that $MRS_{x,y}$ is diminishing?

(b) On the same graph draw a second indifference curve, $U_2 = 200$. Show how $MRS_{x,y}$ depends on x and y, and use this information to determine if $MRS_{x,y}$ is diminishing for this utility function.

LEARNING-BY-DOING EXERCISE 3.4

Indifference Curves with Increasing $MRS_{x,y}$

Problem Suppose a consumer's preferences between two goods (x and y) can be represented by the utility function $U = Ax^2 + By^2$, where A and B are positive constants. For this utility function $MU_x = 2Ax$ and $MU_y = 2By$. Show that $MRS_{x,y}$ is increasing.

Special Functional Forms

- A consumer's willingness to substitute one good for another will depend on the goods in question.
- For example, one consumer may view Coke and Pepsi as perfect substitutes, right shoes and left shoes as perfect complements, etc.

Special Functional Forms

Perfect Substitutes: U = Ax + By

Where: A and B are positive constants.

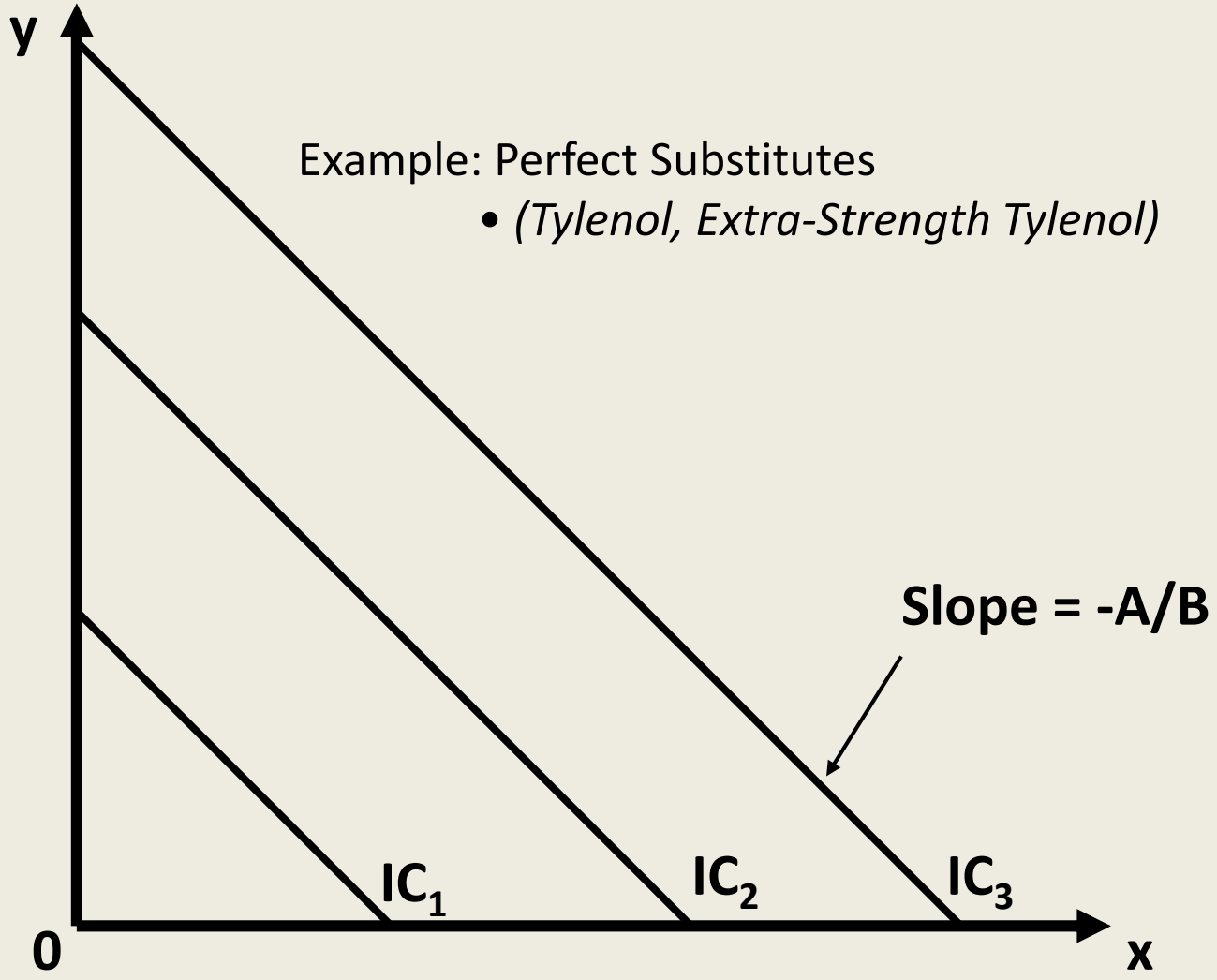
MUx = A MUy = B MRSx,y = A/B (constant)

For example, let A = 2 and B = 1.
MUx = 2 MUy = 1 MRSx,y = 2

This means that each X always gives twice MU of each Y.

Therefore, the consumer is always willing to give up
2 units of Y for 1 more unit of X.

Special Functional Forms



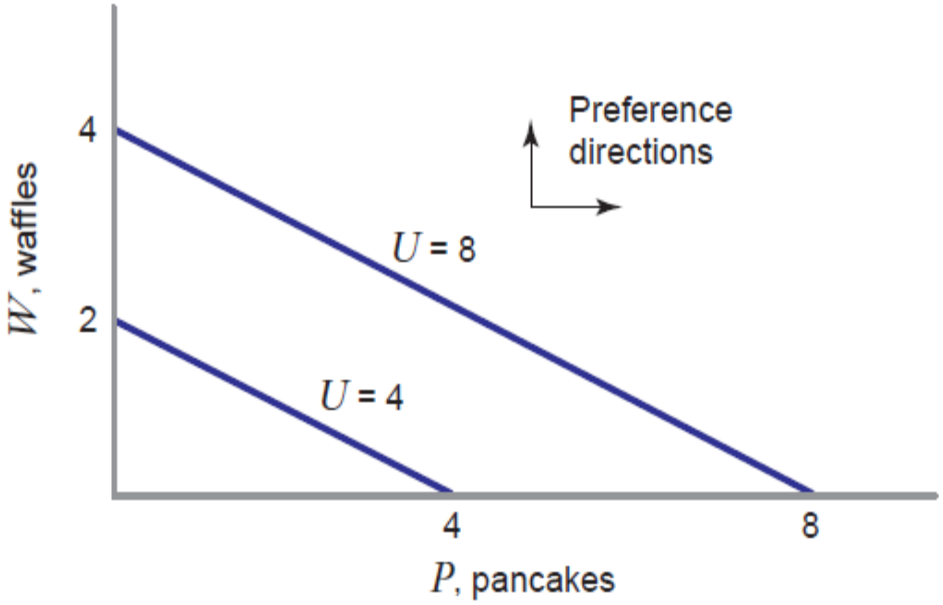


FIGURE 3.13 **Indifference Curves with Perfect Substitutes**

A consumer with the utility function $U = P + 2W$ always views two pancakes as a perfect substitute for one waffle. $MRS_{P,W} = 1/2$, and so indifference curves are straight lines with a slope of $-1/2$.

Special Functional Forms

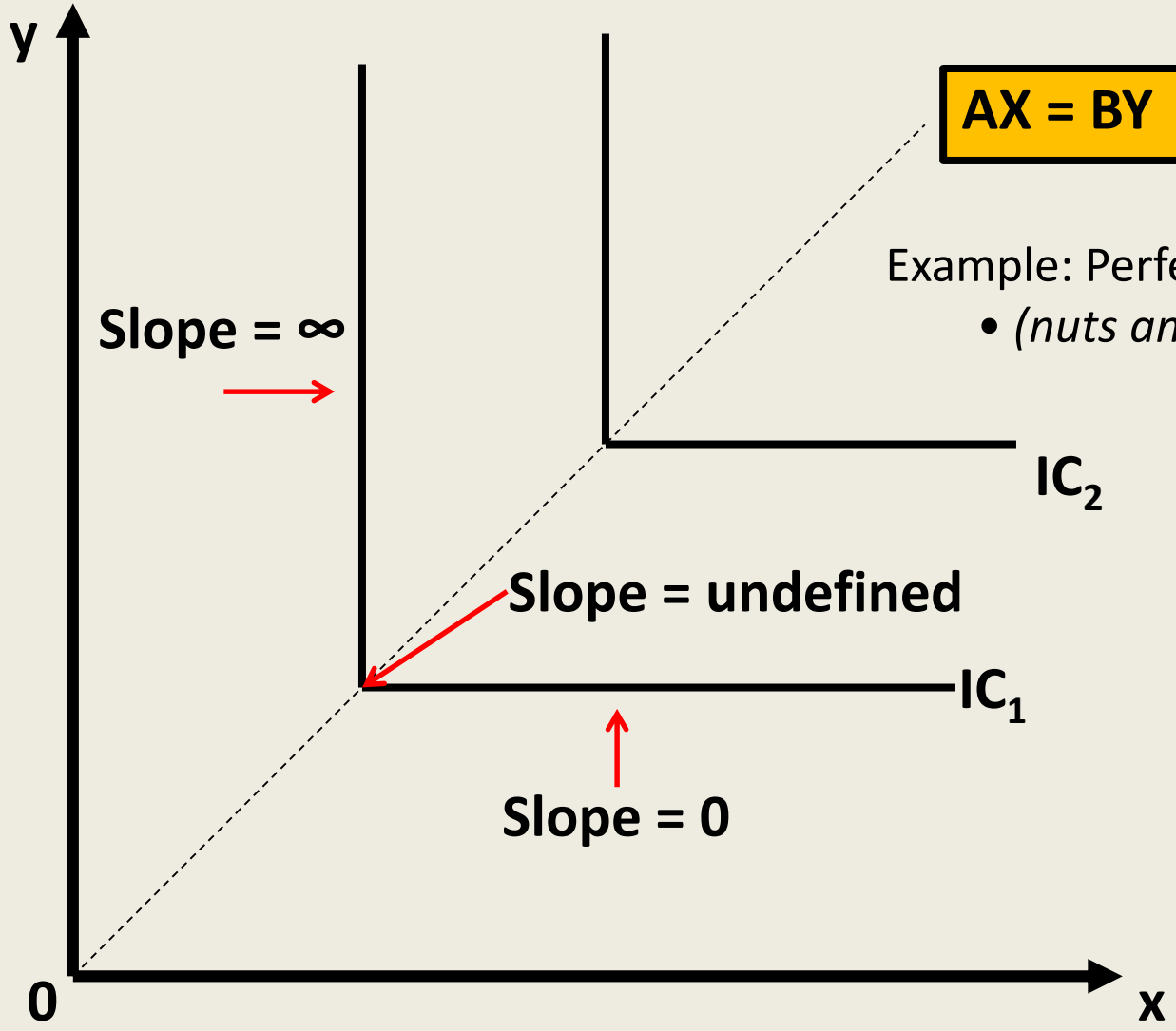
Perfect Complements: U = min(AX, BY)

Where: A and B are positive constants.

MUx = 0 or A
MUy = 0 or B

MRSx,y is 0 or infinite or undefined (corner)

Special Functional Forms



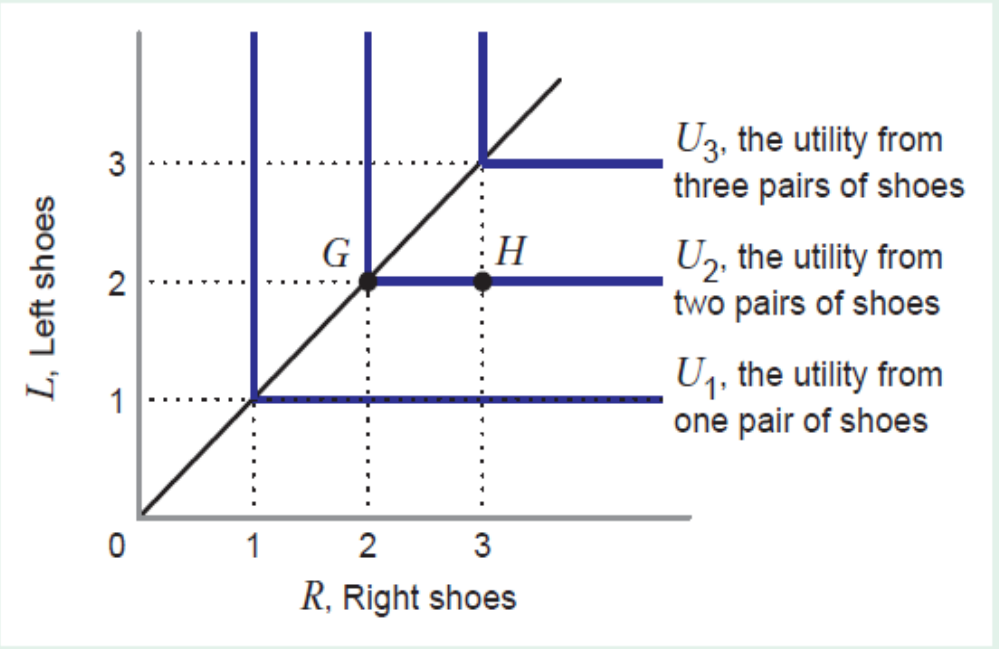


FIGURE 3.14 **Indifference Curves with Perfect Complements**

The consumer wants exactly one left shoe for every right shoe. For example, his utility at basket *G*, with 2 left shoes and 2 right shoes, is not increased by moving to basket *H*, containing 2 left shoes and 3 right shoes.

Special Functional Forms

Cobb-Douglas: $U = Ax^{\alpha}y^{\beta}$

where: A, α, β are positive constants.

$MU_X = \alpha Ax^{\alpha-1}y^{\beta}$

$MU_Y = \beta Ax^{\alpha}y^{\beta-1}$

$MRSx,y = (\alpha y)/(\beta x)$

Special Functional Forms

WHY Cobb-Douglas: $U = Ax^{\alpha}y^{\beta}$??

$MU_X = \alpha Ax^{\alpha-1}y^{\beta}$ $MU_Y = \beta Ax^{\alpha}y^{\beta-1}$

MUx and MUy are positive >>> **more is better**

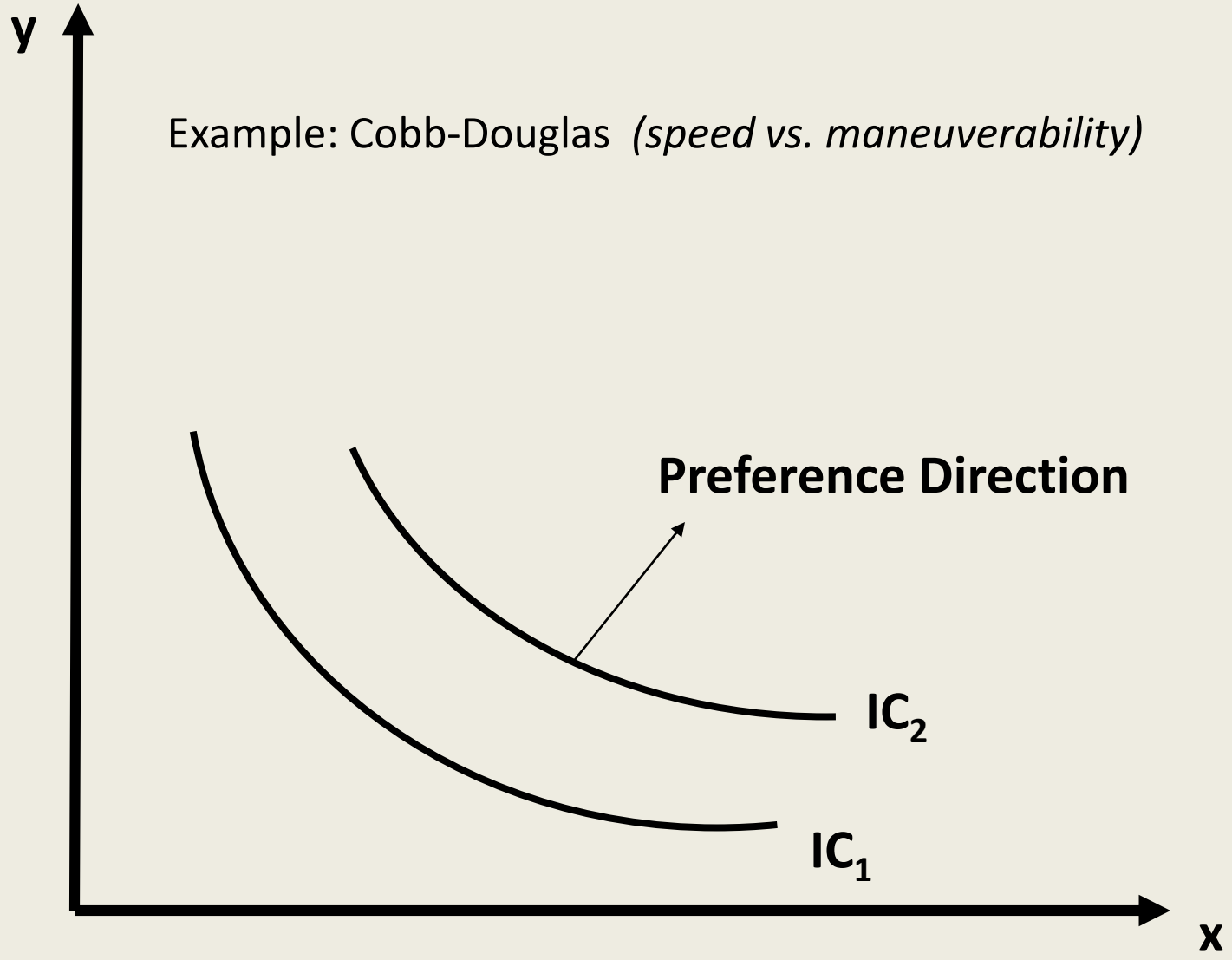
MUx is decreasing in X >>> **diminishing MUx**

MUy is decreasing in Y >>> **diminishing MUy**

That is, Cobb-Douglas satisfies two assumptions on Utility Function.

Special Functional Forms

Example: Cobb-Douglas *(speed vs. maneuverability)*



Special Functional Forms

Quasi-Linear Preferences:

U = v(x) + Ay

i.e. sum of a linear function and a non-linear function

Where: A is a positive constant.

$$MU_x = dv(x)/dx > 0$$
$$MU_y = A$$

Special Functional Forms

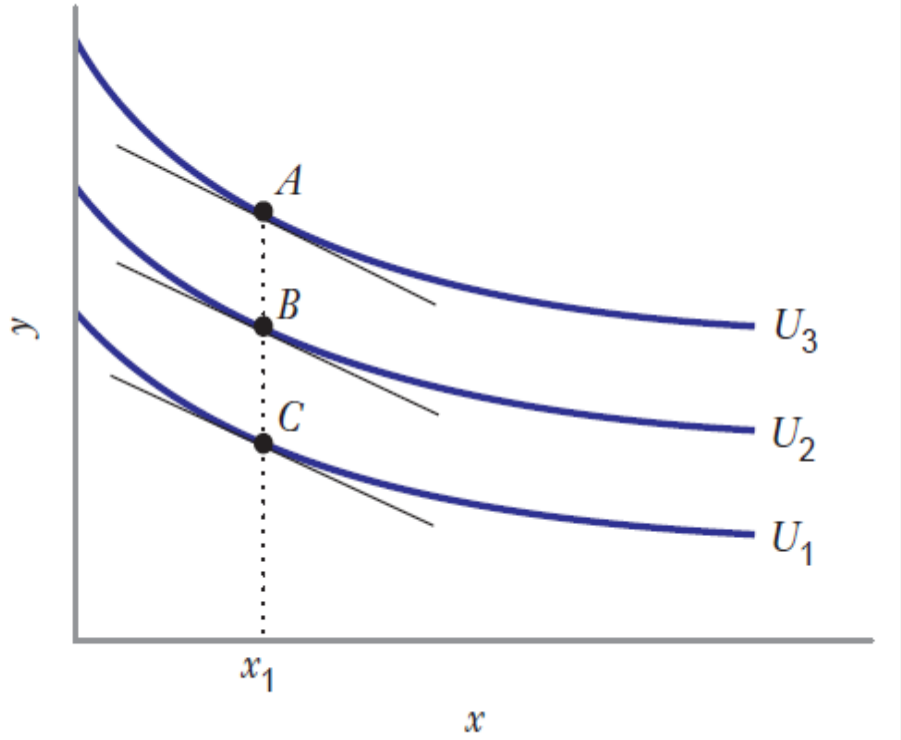


FIGURE 3.15 **Indifference Curves for a Quasilinear Utility Function**

A quasilinear utility function has the form $U(x, y) = v(x) + by$, where $v(x)$ is a function that increases in x and b is a positive constant. The indifference curves are parallel, so for any value of x (such as x_1), the slopes of the indifference curves will be the same (e.g., the slopes of the indifference curves are identical at baskets A, B, and C).

3.3. Jimmy has the following utility function for hot dogs: $U(H) = 10H - H^2$, with $MU_H = 10 - 2H$.

a) Plot the utility and marginal utility functions on two separate graphs.

b) Suppose that Jimmy is allowed to consume as many hot dogs as he likes and that hot dogs cost him nothing. Show, both algebraically and graphically, the value of H at which he would stop consuming hot dogs.

3.4. Consider the utility function $U(x, y) = y\sqrt{x}$ with the marginal utilities $MU_x = y/(2\sqrt{x})$ and $MU_y = \sqrt{x}$.

a) Does the consumer believe that more is better for each good?

b) Do the consumer's preferences exhibit a diminishing marginal utility of x? Is the marginal utility of y diminishing?

3.13. Draw indifference curves to represent the following types of consumer preferences.

a) I like both peanut butter and jelly, and always get the same additional satisfaction from an ounce of peanut butter as I do from 2 ounces of jelly.

b) I like peanut butter, but neither like nor dislike jelly.

c) I like peanut butter, but dislike jelly.

d) I like peanut butter and jelly, but I only want 2 ounces of peanut butter for every ounce of jelly.