Assignment - Solution
Simultaneous Equations Model

Export-Import Equations System

$$\ln Ex_t = \beta_{10} + \beta_{11} \ln Im_t + \beta_{12} \ln P_{Ext} + \beta_{13} \ln Fx_t + \beta_{14} \ln GDP_{wt} + \varepsilon_{1t} \quad (1)$$

$$\ln Im_t = \beta_{20} + \beta_{21} \ln Ex_t + \beta_{22} \ln Fx_t + \beta_{23} \ln Inf_t + \varepsilon_{2t} \quad (2)$$

where: $\ln Ex_t$ = Natural log of export at time *t*
$\ln Im_t$ = Natural log of Import at time *t*
$\ln P_{Ext}$ = Natural log of Export price at time *t*
$\ln Fx_t$ = Natural log of Exchange rate at time *t*
$\ln GDP_{wt}$ = Natural log of World GDP at time *t*
$\ln Inf_t$ = Natural log of Inflation at time *t*

Endogenous variables in this system include $\ln Ex_t$ and $\ln Im_t$
Exogenous variables in this system include $\ln P_{Ext}$, $\ln Fx_t$, $\ln GDP_{wt}$, and $\ln Inf_t$

1. State reduce form model of these system models.
2. Estimate reduce form model using OLS and prediction of the endogenous variables.
3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.
4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?

1. State reduce form model of these system models.

$$\ln Ex_t = \pi_{10} + \pi_{11} \ln P_{Ext} + \pi_{12} \ln Fx_t + \pi_{13} \ln GDP_{wt} + \pi_{14} \ln Inf_t + w_{1t}$$
$$\ln Im_t = \pi_{20} + \pi_{21} \ln P_{Ext} + \pi_{22} \ln Fx_t + \pi_{23} \ln GDP_{wt} + \pi_{24} \ln Inf_t + w_{2t}$$

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

```
. reg lnEx lnPx lnFx lnGDP lnInf

      Source |       SS       df       MS              Number of obs =      37
-------------+------------------------------           F(  4,    32) =   66.92
       Model |  12.8567881     4  3.21419703           Prob > F      =  0.0000
    Residual |   1.5370373    32  .048032416           R-squared     =  0.8932
-------------+------------------------------           Adj R-squared =  0.8799
       Total |  14.3938254    36  .399828484           Root MSE      =  .21916

------------------------------------------------------------------------------
        lnEx |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnPx |   .3233985   .1490153     2.17   0.038     .0198642    .6269328
        lnFx |  -.4574346   .2424967    -1.89   0.068    -.9513843    .0365151
       lnGDP |   1.297045   .1301112     9.97   0.000     1.032018    1.562073
       lnInf |  -.2376303   .1754565    -1.35   0.185    -.5950235    .1197629
       _cons |   .9587734   1.468087     0.65   0.518    -2.031621    3.949168
------------------------------------------------------------------------------

. predict lnExhat, xb

. reg lnIm lnPx lnFx lnGDP lnInf

      Source |       SS       df       MS              Number of obs =      37
-------------+------------------------------           F(  4,    32) =   20.26
       Model |  24.1162307     4  6.02905768           Prob > F      =  0.0000
    Residual |  9.52343322    32  .297607288           R-squared     =  0.7169
-------------+------------------------------           Adj R-squared =  0.6815
       Total |  33.6396639    36  .934435109           Root MSE      =  .54553

------------------------------------------------------------------------------
        lnIm |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnPx |  -.0040172   .3709245    -0.01   0.991    -.7595656    .7515313
        lnFx |   -1.72784   .6036156    -2.86   0.007    -2.957365   -.4983155
       lnGDP |  -.5231396   .3238689    -1.62   0.116    -1.182839    .1365598
       lnInf |  -.3201353   .4367411    -0.73   0.469    -1.209748    .5694772
       _cons |     26.363   3.654317     7.21   0.000      18.9194     33.8066
------------------------------------------------------------------------------

. predict lnImhat, xb
```

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model.

```
. reg lnEx lnImhat lnPx lnFx lnGDP

      Source |       SS       df       MS              Number of obs =      37
-------------+------------------------------           F(  4,    32) =   66.92
       Model |  12.8567874     4  3.21419685           Prob > F      =  0.0000
    Residual |    1.537038    32  .048032438           R-squared     =  0.8932
-------------+------------------------------           Adj R-squared =  0.8799
       Total |  14.3938254    36  .399828484           Root MSE      =  .21916

------------------------------------------------------------------------------
        lnEx |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnImhat |   .7422783   .5480703     1.35   0.185    -.3741043    1.858661
        lnPx |   .3263801   .1498666     2.18   0.037     .0211117    .6316484
        lnFx |    .825103   1.069916     0.77   0.446    -1.354245    3.004451
       lnGDP |    1.68536    .349491     4.82   0.000     .9734704     2.39725
       _cons |  -18.60991   14.48383    -1.28   0.208    -48.11251    10.89268
------------------------------------------------------------------------------
```

```
. reg lnIm lnExhat lnFx lnInf

      Source |       SS       df       MS              Number of obs =      37
-------------+------------------------------           F(  3,    33) =   27.65
       Model |  24.0659823     3  8.02199409           Prob > F      =  0.0000
    Residual |  9.57368167    33  .290111566           R-squared     =  0.7154
-------------+------------------------------           Adj R-squared =  0.6895
       Total |  33.6396639    36  .934435109           Root MSE      =  .53862

------------------------------------------------------------------------------
         lnIm |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnExhat |  -.4785147   .1677628    -2.85   0.007    -.8198307   -.1371987
         lnFx |  -1.845126   .5135224    -3.59   0.001    -2.889896   -.8003571
       lnInf |  -.3660743   .3992639    -0.92   0.366    -1.178383    .4462342
       _cons |   27.95926   2.467356    11.33   0.000     22.93939    32.97913
------------------------------------------------------------------------------
```

4. Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning on the asymptotic property, which model is the most appropriated model? Why?

```
. reg3 (lnEx lnIm lnPx lnFx lnGDP) (lnIm lnEx lnFx lnInf), ols inst(lnPx lnFx lnGDP
lnInf)

Multivariate regression
------------------------------------------------------------------------------
Equation           Obs  Parms        RMSE    "R-sq"     F-Stat        P
------------------------------------------------------------------------------
lnEx                37      4    .1803998    0.9276     102.57   0.0000
lnIm                37      3    .5859579    0.6632      21.66   0.0000
------------------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnEx         |
         lnIm |    .245524   .0579727     4.24   0.000     .1297445    .3613034
         lnPx |   .2730116   .1136106     2.40   0.019     .0461156    .4999077
         lnFx |  -.1237281   .2129469    -0.58   0.563    -.5490123    .3015562
        lnGDP |   1.389122    .107585    12.91   0.000      1.17426    1.603984
        _cons |  -5.549768   1.944813    -2.85   0.006    -9.433827    -1.66571
-------------+----------------------------------------------------------------
lnIm         |
         lnEx |  -.2257254   .1702545    -1.33   0.190    -.5657471    .1142962
         lnFx |  -2.100195    .554702    -3.79   0.000    -3.208011   -.9923784
        lnInf |  -.3362366   .4342847    -0.77   0.442    -1.203563    .5310899
        _cons |   25.30874   2.594182     9.76   0.000      20.1278    30.48968
------------------------------------------------------------------------------

. reg3 (lnEx lnIm lnPx lnFx lnGDP) (lnIm lnEx lnFx lnInf), 2sls inst(lnPx lnFx
lnGDP lnInf)

Two-stage least-squares regression
------------------------------------------------------------------------------
Equation           Obs  Parms        RMSE    "R-sq"     F-Stat        P
------------------------------------------------------------------------------
lnEx                37      4    .3274401    0.7616      29.98   0.0000
lnIm                37      3    .6052138    0.6407      21.90   0.0000
------------------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnEx         |
         lnIm |    .742281    .818843     0.91   0.368    -.8930613    2.377623
         lnPx |   .3263804   .2239079     1.46   0.150    -.1207946    .7735554
         lnFx |   .8251084   1.598506     0.52   0.607    -2.367328    4.017545
        lnGDP |   1.685362   .5221561     3.23   0.002     .6425441     2.72818
        _cons |  -18.60998   21.63953    -0.86   0.393    -61.82711    24.60714
-------------+----------------------------------------------------------------
lnIm         |
         lnEx |  -.4785146   .1885046    -2.54   0.014    -.8549843   -.1020449
         lnFx |  -1.845126   .5770131    -3.20   0.002    -2.997501   -.6927517
        lnInf |  -.3660742   .4486279    -0.82   0.417    -1.262046    .5298976
        _cons |   27.95926   2.772414    10.08   0.000     22.42236    33.49615
```

```
------------------------------------------------------------------------------
Endogenous variables:  lnEx lnIm
Exogenous variables:   lnPx lnFx lnGDP lnInf
------------------------------------------------------------------------------

. reg3 (lnEx lnIm lnPx lnFx lnGDP) (lnIm lnEx lnFx lnInf), 3sls inst(lnPx lnFx
lnGDP lnInf)

Three-stage least-squares regression
----------------------------------------------------------------------
Equation          Obs  Parms        RMSE    "R-sq"       chi2        P
----------------------------------------------------------------------
lnEx               37      4    .3421059    0.6992     138.86   0.0000
lnIm               37      3    .5715641    0.6407      73.67   0.0000
----------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnEx         |
        lnIm |   .8276964   .7297008     1.13   0.257    -.6024909    2.257884
        lnPx |   .3875424   .1379849     2.81   0.005     .1170969    .6579879
        lnFx |   .9316971   1.461526     0.64   0.524    -1.932842    3.796236
       lnGDP |   1.769392   .4357714     4.06   0.000      .915296    2.623488
       _cons |  -21.32074   18.90018    -1.13   0.259    -58.36442    15.72293
-------------+----------------------------------------------------------------
lnIm         |
        lnEx |  -.4785146   .1780238    -2.69   0.007    -.8274349   -.1295943
        lnFx |  -1.845126   .5449313    -3.39   0.001    -2.913172   -.7770806
       lnInf |  -.3660742   .4236843    -0.86   0.388     -1.19648    .4643318
       _cons |   27.95926   2.618269    10.68   0.000     22.82755    33.09097
------------------------------------------------------------------------------
Endogenous variables:  lnEx lnIm
Exogenous variables:   lnPx lnFx lnGDP lnInf
------------------------------------------------------------------------------

. reg3 (lnEx lnIm lnPx lnFx lnGDP) (lnIm lnEx lnFx lnInf), 3sls ireg3 inst(lnPx
lnFx lnGDP lnInf)

Iteration 1:   tolerance =   .1382335
Iteration 2:   tolerance =  .02305974
Iteration 3:   tolerance =  .00427982
Iteration 4:   tolerance =  .00080918
Iteration 5:   tolerance =  .00015352
Iteration 6:   tolerance =  .00002915
Iteration 7:   tolerance =  5.534e-06
Iteration 8:   tolerance =  1.051e-06
Iteration 9:   tolerance =  1.997e-07

Three-stage least-squares regression, iterated
----------------------------------------------------------------------
Equation          Obs  Parms        RMSE    "R-sq"       chi2        P
----------------------------------------------------------------------
lnEx               37      4    .3511994    0.6829     104.53   0.0000
lnIm               37      3    .5715641    0.6407      73.67   0.0000
----------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
lnEx         |
        lnIm |    .847716   .8361021     1.01   0.311    -.7910139    2.486446
        lnPx |   .4018776   .1435903     2.80   0.005     .1204458    .6833094
        lnFx |   .9566794   1.681353     0.57   0.569    -2.338712    4.252071
       lnGDP |   1.789087   .4936624     3.62   0.000     .8215268    2.756648
       _cons |  -21.95609    21.5847    -1.02   0.309    -64.26133    20.34916
-------------+----------------------------------------------------------------
lnIm         |
        lnEx |  -.4785146   .1780238    -2.69   0.007    -.8274349   -.1295943
        lnFx |  -1.845126   .5449313    -3.39   0.001    -2.913172   -.7770806
       lnInf |  -.3660742   .4236843    -0.86   0.388     -1.19648    .4643318
       _cons |   27.95926   2.618269    10.68   0.000     22.82755    33.09097
------------------------------------------------------------------------------
Endogenous variables:  lnEx lnIm
Exogenous variables:   lnPx lnFx lnGDP lnInf
------------------------------------------------------------------------------
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