Chapter 7 Curve Sketching

In this chapter we will learn how to sketch the curve of relation by considering its properties such as the intercepts, symmetrical, domain, range, and asymptotes.

Guidelines for Curve Sketching

1. **Domain** (D) **and Range** (R)

   Domain is the set of values of x for which $f(x)$ is defined.

   Range is the set of values of $f(x)$ correspond to the domain.

2. **Intercepts**

   To find *y*-intercept, we set $x = 0$ and solve for *y*.

   To find *x*-intercept, we set $y = 0$ and solve for *x*.

3. **Symmetry**

   (i) If $f(-x) = f(x)$ for all x in D, then *f* is an even function and the curve is symmetric about the *y*-axis (reflectional symmetry).

   (ii) If $f(-x) = -f(x)$ for all x in D, then *f* is an odd function and the curve is symmetric about the origin (rotational symmetry).

4. **Asymptotes**

   Method I (Arranging the relation)

   (i) *Horizontal Asymptotes*.

   To find horizontal asymptotes, we arrange the relation in the form of $x = g(y)$. Next we find the real number *b* and substitute *b* into *y* to have the denominator $g(y) = 0$, then the line $y = b$ the horizontal asymptote of the curve.

   (ii) *Vertical Asymptotes*.

   To find vertical asymptotes, we arrange the relation in the form of $y = f(x)$ Next we find the real number *c* and substitute *c* into *x* to have the denominator $f(x) = 0$, then the line $x = c$ the vertical asymptote of the curve.

   Method II (Using Limit)

   (i) *Horizontal Asymptotes*.

   Consider the function $y = f(x)$. If either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$, then the line $y = b$ is a horizontal asymptote of the curve $y = f(x)$.

   (ii) *Vertical Asymptotes*.

   Consider the function $x = g(y)$. If either $\lim_{y\to\infty} g(y) = c$ or $\lim_{y\to-\infty} g(y) = c$, then the line $x = c$ is a vertical asymptote of the curve $y = f(x)$.

Method III (Arranging the relation in the form of a product of linear factors)

We can use method to find not only the horizontal and vertical asymptotes but also different type of asymptotes as well. To find the asymptotes, we arrange the relation in the form of the product of linear factors on one side and a constant on the other side, then the equation of the asymptote can be found by setting each linear factor equal to zero.

Example 7.1: Consider the relation $x^2y - x^2 - y + 4 = 0$. Find the domain, range, intercepts, symmetry, asymptotes, and sketch the curve of this relation.

Example 7.2: Consider the relation $x^2y + 2y = 6$. Find the domain, range, intercepts, symmetry, asymptotes, and sketch the curve of this relation.

Example 7.3: Consider the relation $x^2 - xy - x + y - 2 = 0$. Find the domain, range, intercepts, symmetry, asymptotes, and sketch the curve of this relation.