FN 312
Midterm Solutions

Question 1

a) To construct an equal-weighted portfolio, we need an equal number of dollars in each company. Since we have $10,000 to invest and there are two companies, we need to invest $5000 in each company's stock. We will purchase 5,000/100 = 50 shares of the first company and $5000/50=100 shares of the second company.

   To construct a value-weighted portfolio we need to purchase an equal percentage of each company's market capitalization. The market capitalization of the first is 10000*100 =1,000,000 and the market cap of the second is 60000*50 = 3,000,000. If x is the equal percentage of each company's market capitalization that we will invest in, then 1,000,000x + 3,000,000x = 10,000. Solve to get x = 1/400 so we purchase 1,000,000/400 = $2,500 of the first company's stock and 3,000,000/400 = $7500 of the second company's stock. This is 2,500/100 = 25 shares of the first company and 7500/50 = 150 shares of the second company.

b) The (net simple) return of company 1's shares is (150-100)/100 = 0.5 and the return of company 2's shares is 0. Now we apply the idea that the portfolio return is the weighted average of its individual stock returns where the weights are the shares of wealth invested in each stock. The equal-weighted portfolio return is 0.5*.5 + 0.5*0 = 25% while the value weighted return is 0.25*.5+.75*0 = 12.5%. The equal-weighted portfolio has a higher return because it held a greater share of wealth in the stock with the higher return.

c) To keep the equal weighted portfolio equally weighted you must sell some of the first company's shares and buy more of the second company's shares. Since the first company's stock price appreciated and the second company's did not, the wealth share held in the first company's stock before rebalancing is greater than 50%

   To keep the value-weighted portfolio value-weighted nothing must be done. Since the number of shares outstanding hasn't changed for either company, the relative market capitalization has changed in proportion to the change in the share prices. The relative wealth share held in each company has also changed in proportion to the change in share price.

Question 2

a)

Mean return on portfolio = $R_f + (R_p - R_f)y$

$= 7\% + (17\% - 7\%)y = 7\% + 10\%y$

If the mean of the portfolio is equal to 15%, then solving for y we will get:

$15\% = 7\% + 10\%y \quad => \quad y = (15\% - 7\%)/10\% \quad => \quad y = 0.8$

Thus, in order to obtain a mean return of 15%, the client must invest 80% of total funds in the risky portfolio and 20% in Treasure bills.

Investment proportions of the client's funds:

- 20% in T-bills
- 0.8 x 27% = 21.6% in Stock A
- 0.8 x 33% = 26.4% in Stock B
- 0.8 x 40% = 32.0% in Stock C

b)

Portfolio standard deviation = y x 27%. If your client wants a standard deviation of 20%, then

$y = (20\%/27\%) = 0.7407 = 74.07\%$ in the risky portfolio.

Mean return = $7\% + (17\% - 7\%)y = 7\% + 10\%\ (0.7407) = 7\% + 7.407\% = 14.407\%$.

c)

$y^* = (R_p - R_f)/0.01A\sigma^2 \quad => \quad y^* = (17 - 7)/(0.01 \times 3.5 \times 27^2) = 10/25.515 = 0.3919$

Since I invest more in the risk free asset compared to the risky portfolio so I am a relatively risk averse investor.

d)

The slope of the CML = (13% - 7%)/25% = 0.24. You can draw a graph as we did in class. My fund allows an investor to achieve a higher mean for any given standard deviation than would a passive strategy, that is, a higher expected return for any given level of risk. Indeed, my fund's reward-to-variability ratio is (17% - 7%)/27% = 0.37, which is much better than 0.24 (the RTRR of the S&P 500).

e)

The fee would reduce the reward-to-variability ratio, that is, the slope of the CAL. Clients will be indifferent between your fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the maximum fee. In this case:

Slope of CAL with fee = (17% - 7% - f)/27 = (10 - f)/27%.

Slope of CML (which requires no fee) = (13% - 7%)/25% = 0.24

Setting these slopes equal, then we get:

(10% - f)/27% = 0.24 => 10% - f = 27% x 0.24 => f = 10% - 6.48% = 3.52% per year.

Question 3

a) Yes. A risky asset whose return is negatively correlated with the market's return will have a negative beta. According to the expected return-beta representation of CAPM this risky asset will have a lower expected return than the risk free rate.a)

b) Not necessarily. When beta=1, this implies Cov(ri, rm)=Var(rm).
Recall that the correlation corr(ri,rm) = Cov(ri,rm)/sigma(ri) sigma(rm).
Thus the correlation is not one unless sigma(rm)=sigma(ri).