Question 1 (15 points)

a) (5 points) Suppose you discover a treasure chest of $10 billion in cash. Is this a real or financial asset? Can this discovery contribute to the productive capacity of the economy?

Cash is essentially a financial asset because it is the liability of the government. Financial assets do not represent a society's wealth.

The cash does not directly contribute to the productive capacity of the economy.

b) (5 points) What would you expect to happen to the spread between yields on commercial paper and Treasury bills if the economy were to enter a steep recession? Explain your answer.

Commercial paper is subject to default risk whereas Treasury bills are not. The spread between yields on commercial paper and T-bills would therefore be expected to widen, insofar as a deterioration in the economy increases credit risk, that is, the likelihood of default.

c) (5 points) Suppose security X is a 3 month bill selling at $9,674 and security Y is a 6-month bill selling at $9,539. Both have a face value of $10,000. Which of the two securities have a higher effective annual yield?

There are four 3-month periods in a year and two 6-month periods in a year

Security X offers an effective annual yield of

$$\left(\frac{10000}{9674}\right)^4 - 1 = 14.17\%$$

and security Y offers an effective annual yield of

$$\left(\frac{10000}{9539}\right)^2 - 1 = 9.9\%$$

Security X gives the higher effective annual yield.

Question 2 (20 points)

You are given the following information about possible investments

Asset	Mean E(r)	Standard Deviation	Correlation with market
Technology Stocks	18%	30%	1
Consumer Utility Stocks		20%	0.5
Gold		20%	-0.5
T-bills	6%	0%	0

a) (7 points) If the market standard deviation is 20% what are the CAPM betas of each of these assets? Briefly discuss whether the values of beta for each asset make sense in terms of its risk characteristics.

$\beta = \frac{Cov(r_i, r_m)}{\sigma_m^2} = \frac{\rho_{im}\sigma_i}{\sigma_m}$

The technology stock beta is $\frac{1 * 0.3}{0.2} = 1.5$

4 consumer utility stock beta is $\frac{0.5 * 0.2}{0.2} = 0.5$

Gold beta is $\frac{-0.5 * 0.2}{0.2} = -0.5$

T-bill beta is 0

The values of beta makes sense as T-bills have no risk, technology stocks have the largest systemic risk

3 followed by consumer utility stocks since they are stable and are a necessity. Gold is typically thought of as a hedge because it's price moves opposite to the market and hence has negative beta.

b) (7 points) Assume all assets are priced correctly according to CAPM. What are the expected returns of the market, consumer utility stocks, and gold?

For ~~consumer~~ technology stocks:

3 $0.18 = 0.06 + 1.5(E(r_m) - 0.06)$

$\Rightarrow E(r_m) = 14\%$

For consumer utility stocks

2 $E(r) = 0.06 + 0.5(0.14 - 0.06) = 0.10$

2 For gold $E(r) = 0.06 - 0.5(0.14 - 0.06) = 0.02$

c) (6 points) In addition to the assets described above, you are told that the expected return on ABC, Inc. equity shares is -14% and its beta is -2. Did the market correctly price this firm? If not, explain whether it is over or undervalued and outline your investment strategy to take advantage of this mispricing.

ABC's expected return is −14% while the CAPM predicts that the expected return should be

3

$0.06 - 2(0.14 - 0.06) = -0.10$

– Thus, the expected return is too low implying that

3 ABC is overpriced. (this means their price is too high compared to the fair CAPM value

– To take advantage of this mispricing we should sell ABC's shares

Question 3 (35 points)

Consider the following two assets:

- Asset A's expected return is 5.5% and return standard deviation is 31%
- Asset B's expected return is 3% and return standard deviation is 55%

The correlation between assets A and B is 0.2. The risk-free rate is 2%.

The table below indicates the expected return and the return standard deviation for portfolios that put weight w on asset A and weight 1-w on asset B.

Weight	Expected Return	Return Standard Deviation
w=0.9	5.25%	29.49%
w=0.5	4.25%	34.16%
w=0.2	3.5%	45.65%

a) (5 points) Complete the above table.

Return stdev for w=0.9 $= \sqrt{0.9^2(\cancel{0.055}\ 0.31^2) + 0.1^2(0.55^2) + 2(0.2)(0.9)(0.1)(0.31)(0.55)}$

$= \sqrt{0.077841 + 0.003025 + \cancel{0.0682}\ 0.006138} = \sqrt{\cancel{0.149066}\ 0.087004}$

$= \cancel{0.386}\ 0.2949$

Solving for w

$0.035 = w(0.055) + (1-w)(0.03)$

$0.035 = 0.055w + 0.03 - 0.03w$

$0.005 = 0.025w$

$w = 0.2$

b) (7 points) Set up the optimization problem and find the portfolio weights for the global minimum variance portfolio.

$$\min_{w} \; w^2 0.31^2 + (1-w)^2 0.55^2 + 2(w)(1-w)(0.2)(0.31)(0.55)$$

$$w^2 0.0961 + (1-2w+w^2)\,0.3025 + 0.0682w - 0.0682w^2$$

$$w^2 0.0961 + 0.3025 - 0.605w + 0.3025w^2 + 0.0682w - 0.0682w^2$$

$$\overset{0.3025}{\cancel{0.2986}} + \overset{0.3304}{\cancel{0.2343}}\, w^2 - 0.5368\,w$$

FOC $\overset{0.6608}{\cancel{0.4686}}\, w = 0.5368$

$$w = 0.8123$$

OR if kept generalized:

$$\min_{w} \; w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\sigma_A\sigma_B\rho$$

$$w^2\sigma_A^2 + \sigma_B^2 - 2w\sigma_B^2 + w^2\sigma_B^2 + 2w\sigma_A\sigma_B\rho - 2w^2\sigma_A\sigma_B\rho$$

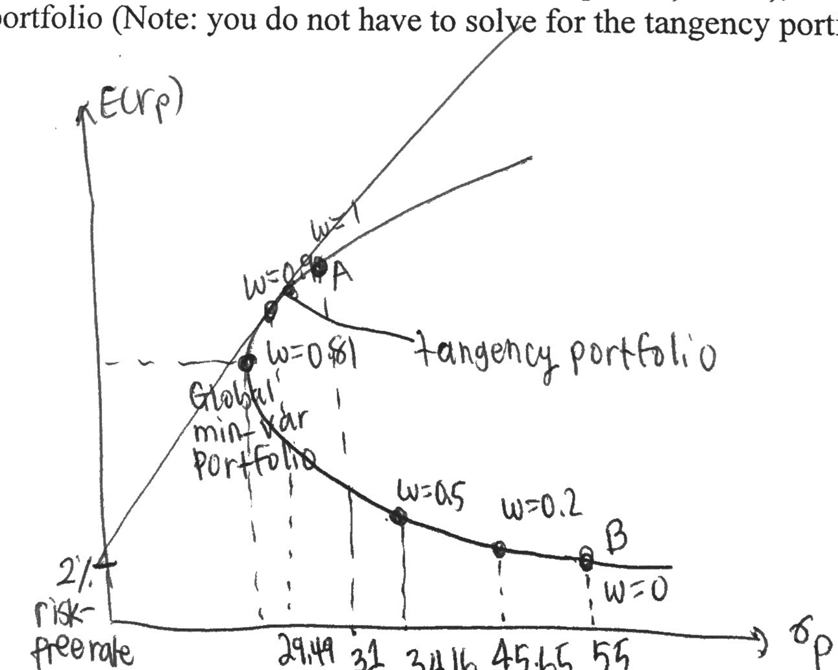
FOC $2w\sigma_A^2 - 2\sigma_B^2 + 2w\sigma_B^2 + 2\sigma_A\sigma_B\rho - 4w\sigma_A\sigma_B\rho$

$$\sigma_A^2 w + \sigma_B^2 w - 2w\sigma_A\sigma_B\rho = \sigma_B^2 - \sigma_A\sigma_B\rho$$

$$w = \frac{\sigma_B^2 - \sigma_A\sigma_B\rho}{\sigma_A^2 + \sigma_B^2 - 2\sigma_A\sigma_B\rho} = \frac{0.55^2 - (0.31)(0.55)(0.2)}{0.31^2 + 0.55^2 - 2(0.31)(0.55)(\ldots)}$$

$$= \frac{0.3025 - 0.0341}{0.0961 + 0.3025 - 0.0682} = \frac{0.2684}{0.3304} = 0.8123$$

c) (7 points) Draw a careful sketch of the minimum variance frontier. Clearly provide labels for assets A and B, the portfolios from parts a) and b), and the tangency portfolio (Note: you do not have to solve for the tangency portfolio.).



d) (5 points) Explain why the market portfolio is the tangency portfolio.

This is because all investors are mean variance optimizers and we know by the two fund seperation theorm that they hold various combinations of the risk free asset and the tangency portfolio.

Because the market supply of risky assets must equal the market demand for assets, the market portfolio must be the tangency portfolio.

e) (5 points) Suppose an investor with degree of risk aversion A=5 chooses the portfolio with w=0.5 as her risky portfolio. How much will she then decide to invest in the risky portfolio versus risk-free asset? Do you think that this investor is relatively risk averse? Explain.

$$y^* = \frac{E(r_p) - r_f}{A\sigma_p^2} = \frac{4.25 - 2}{5(34.16)^2} = 0.0385$$

Yes, this investor is relatively risk averse as she invests the majority of her funds in the risk-free asset.

f) (6 points) Is the portfolio in part e) the optimal risky portfolio? Given that there is an investor that chooses to invest in this portfolio, do you think the two fund separation theorem holds? If not, explain why this investor's choice of risky portfolio may be different from others.

NO. The optimal risky portfolio is the tangency portfolio with the highest Sharpe ratio. In this case, the two fund seperation theorm does not hold since the theorm states that all investors must choose to hold the same optimal risky portfolio

This investor's choice of risky portfolio may be different from others if the information set she has is different from other investors.