Tutorial – Matrix (Supplement)

1. If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 7 & 0 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, compute

(i) $3\mathbf{A}^T + \mathbf{D}$ Ans: $\begin{bmatrix} 4 & 2 & 20 \\ 7 & -3 & 2 \end{bmatrix}$

(ii) $(\mathbf{B} - \mathbf{C})^T$ Ans: $\begin{bmatrix} 0 & 3 \\ 3 & -3 \end{bmatrix}$

(iii) $2\mathbf{B}^T - 3\mathbf{C}^T$ Ans: $\begin{bmatrix} -1 & 5 \\ 6 & -8 \end{bmatrix}$

(iv) $2\mathbf{B} + \mathbf{B}^T$ Ans: $\begin{bmatrix} 3 & 10 \\ 11 & -3 \end{bmatrix}$

(v) $\mathbf{C}^T - \mathbf{D}$ Ans: undefined

(vi) $(\mathbf{D} - 2\mathbf{A}^{T})^{T}$ Ans: $\begin{bmatrix} -1 & -3 \\ 2 & 2 \\ -15 & 2 \end{bmatrix}$

2. If $\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 0 & 0 & -1 \\ 2 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}$ $\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{O} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, compute

(i) $\mathbf{A}^2$ Ans: undefined

(ii) $\mathbf{A}^T\mathbf{A}$ Ans: $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

(iii) $\mathbf{B}^3$ Ans: $\begin{bmatrix} 0 & 0 & -4 \\ 2 & -1 & -2 \\ 0 & 0 & 8 \end{bmatrix}$

(iv) $\mathbf{A}(\mathbf{B}^T)^2\mathbf{C}$ Ans: $\begin{bmatrix} -6 & 3 \\ -4 & 5 \end{bmatrix}$

(v) $(\mathbf{AC})^2$ Ans: $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

(vi) $\mathbf{A}^T(2\mathbf{C}^T)$ Ans: $\begin{bmatrix} 2 & 4 & 0 \\ -2 & -6 & 2 \\ 0 & -2 & 2 \end{bmatrix}$

(vii) $(\mathbf{BA}^T)^T$ Ans: $\begin{bmatrix} 0 & 3 & 0 \\ -1 & -1 & 2 \end{bmatrix}$

(viii) $(2\mathbf{B})^T$ Ans: $\begin{bmatrix} 0 & 4 & 0 \\ 0 & -2 & 0 \\ -2 & 0 & 4 \end{bmatrix}$

(ix) $(2\mathbf{I})^2\text{-}2\mathbf{I}^2$ Ans: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(x) $\mathbf{A}(\mathbf{I}\text{-}\mathbf{O})$ Ans: $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(xi) $\mathbf{I}^T\mathbf{O}$ Ans: $\mathbf{O}$

(xii) $(\mathbf{AB})(\mathbf{AB})^T$ Ans: $\begin{bmatrix} 6 & -7 \\ -7 & 9 \end{bmatrix}$

(xiii) $\mathbf{B}^2\text{-}3\mathbf{B}+2\mathbf{I}$ Ans: $\begin{bmatrix} 2 & 0 & 1 \\ -8 & 6 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

3. If **A** is 2 x 3, **B** is 3 x 1, **C** is 2 x 5, **D** is 4 x 3, **E** is 3 x 2, and **F** is 2 x 3, determine the size of the following matrices:

   AE, **DE**, **EC**, **DB**, **FB**, **BC**, **EA**, **E**(**AE**), **E**(**FB**) and (**F** + **A**)**B**

4. If $\underline{\mathbf{A}}$ is a 4x4 matrix such that $\underline{\mathbf{A}}\begin{bmatrix}0\\6\\2\\4\end{bmatrix}=\begin{bmatrix}2\\0\\0\\-2\end{bmatrix}$ and $\underline{\mathbf{A}}\begin{bmatrix}3\\6\\6\\15\end{bmatrix}=\begin{bmatrix}-12\\0\\6\\3\end{bmatrix}$, what is the product $\underline{\mathbf{A}}\begin{bmatrix}1\\11\\5\\11\end{bmatrix}$?

   (Hints: you can obtain $\underline{\mathbf{A}}\begin{bmatrix}1\\11\\5\\11\end{bmatrix}$ without finding the matrix $\underline{\mathbf{A}}$)   Ans: $\begin{bmatrix}-1\\0\\2\\-2\end{bmatrix}$

5. By using the row operations, solve the linear system

$$2x+3y=-1$$
$$2x+y=5$$
$$x+y=1$$

   Ans: $x=4$ and $y=-3$

6. By using the row operations, solve the linear system

$$x+2y+z-6=0$$
$$3z+y-3=0$$
$$x+y+2z-1=0$$

   Ans: No solution (inconsistent)

7. By using the row operations, solve the linear system

$$2x_1+3x_2+2x_3+6x_4=10$$
$$x_2+2x_3+x_4=2$$
$$3x_1-3x_3+6x_4=9$$

   Ans: Many solutions that satisfied $\begin{cases}x_1+\frac{5}{2}x_4=4\\ x_2=0\\ x_3+\frac{1}{2}x_4=1\end{cases}$

8. By using the row operations, solve the linear system

$$x_1+x_2-x_3+x_4+x_5=0$$
$$x_1+x_2+x_3-x_4+x_5=0$$
$$x_1-x_2-x_3+x_4-x_5=0$$
$$x_1+x_2-x_3-x_4-x_5=0$$

   Ans: Many solutions that satisfied $x_1=0, x_2=-x_5, x_3=-x_5, x_4=-x_5$

9. By using the row operations, solve the linear system

$$x_1 + x_2 + x_3 - x_4 = 0$$
$$x_1 - x_2 - x_3 + x_4 = 0$$
$$x_1 + x_2 - x_3 - x_4 = 0$$
$$x_1 + x_2 - x_3 + x_4 = 0$$

Ans: Many solutions that satisfied $x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0$

10. the following system of linear equations over the real numbers, where x_1, x_2, x_3 are variables also b_1, b_2, b_3 and C are real constants.

$$x_1 + 5x_2 + 6x_3 = b_1$$
$$2x_1 + 4x_2 + 6x_3 = b_2$$
$$2x_1 + 6x_2 + Cx_3 = b_3$$

(a) Write down the matrix equation ($\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{\mathbf{b}}$) and the augmented matrix that represents the above system of linear equations.

(b) By performing row operations, obtain the upper triangle form of <u>**A**</u> .

(c) If $b_1 = 0, b_2 = 0, b_3 = 0$, is it possible that this linear system will have no solution?

(d) If $b_1 = 6, b_2 = 6, b_3 = 8$, is it possible that this linear system will have infinitely many solutions? If possible, give <u>an example</u> of a value of *C*.

(e) Solve the given linear system with your specified value of *C* in (d).

(f) If $b_1 = 6, b_2 = 6, b_3 = 8$, is it possible that this linear system to have a unique solution? If possible, give <u>an example</u> of a value of *C*.

(g) Solve the given linear system with your specified value of *C* in (f).

11. Find the inverse of

(i) $\begin{bmatrix} 6 & 1 \\ 7 & 1 \end{bmatrix}$ Ans: $\begin{bmatrix} -1 & 1 \\ 7 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ Ans: non-singular/not invertable

(iii) $\begin{bmatrix} -5 & 4 & -3 \\ 10 & -7 & 6 \\ 8 & -6 & 5 \end{bmatrix}$ Ans: $\begin{bmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Ans: non-singular/not invertable

12. Compute the determinant of **A**, **B** and **C**

$$|\mathbf{A}| = \begin{vmatrix} 1 & 3 & 2 \\ 8 & 17 & 21 \\ 2 & 7 & 1 \end{vmatrix}$$

$$|\mathbf{B}| = \begin{vmatrix} 1 & -2 & 3 & 0 \\ 0 & 4 & 5 & -6 \\ 1 & 0 & 2 & 3 \\ 4 & 0 & 2 & 3 \end{vmatrix}$$

$$|\mathbf{C}| = \begin{vmatrix} 1 & 2 & 3 & 0 & 0 \\ 1 & 0 & 4 & 2 & 3 \\ 2 & 0 & 1 & 4 & 5 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & 2 & 1 & 2 & 3 \end{vmatrix}$$

Ans: 16, 270, 16

13. Given $\underline{\mathbf{A}} = \begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$; $\underline{\mathbf{B}} = \begin{bmatrix} 4 & 0 & 0 & 3 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 0 \end{bmatrix}$ and

$$\underline{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 11 & 1/6 & 0 & 0 \\ -5 & 1/3 & 1/9 & 0 \\ 4 & 5/2 & 7/9 & 2/3 \end{bmatrix}$$

(a) If $\underline{\mathbf{A}} = \underline{\mathbf{B}}\underline{\mathbf{D}}^3\underline{\mathbf{C}}$, find the determinant of $\underline{\mathbf{D}}$.

Ans: $\mathbf{D} = \frac{3}{2}(1 - t^2)$

(b) Find the solution to $\underline{\mathbf{B}}^{\mathrm{T}}\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.

Ans: $\begin{bmatrix} \frac{1}{4} \\ 1 \\ \frac{11}{4} \\ \frac{1}{2} \end{bmatrix}$