Exercise 1 (Part 3)

1. Determine whether the statement forms are logically equivalent. In each case, construct a truth table to justify your answer.

   (a) $(p \to q) \to (q \to p)$ and $(p \vee q) \to (p \wedge q)$

   (b) $(p \vee q) \wedge \sim (p \wedge q)$ and $p \leftrightarrow q$

   (c) $(p \leftrightarrow q) \leftrightarrow r$ and $p \leftrightarrow (q \leftrightarrow r)$

2. Define the truth table for a connective operator $\oplus$ as follow.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

   Construct a truth table for each of the following statement forms.

   (a) $p \oplus p$ (b) $(p \oplus p) \oplus p$ (c) $(p \oplus p) \oplus (p \oplus p)$

3. Determine whether or not each of the following statements is a tautology, a contradiction, or neither of them.

   (a) $(p \leftrightarrow q) \to p \wedge q$ (b)$\sim (p \leftrightarrow q) \to \sim (p \wedge q)$

4. Let p, q and r be statements such that $(p \leftrightarrow q) \vee (q \to r)$ is **false**. Determine the truth value of $(p \vee q \vee r) \wedge (\sim p \vee \sim q \vee \sim r)$. Explain your answer.

5. Consider the following statement.

   If I do not go to class and I get an A for that class, then I am smart.

   (a) Write the **negation** of the above statement.

   (b) Write the **contrapositive**, **inverse**, and **converse** of the above statement.

6. Use truth tables to determine whether the argument forms are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer.

   (a)
   $p \to \sim q$
   $\sim q \to p$
   $\therefore$ $p \vee \sim q$

   (b)
   If I skip classes and I pass these classes, then I read the text books.
   I do not read the text books or I skip classes.
   $\therefore$ I do not pass these classes.

   (c) (Optional)
   If I skip classes and I pass these classes, then I read the text books.
   I do not read the text books.
   I skip classes.
   $\therefore$ I do not pass these classes.

(d) (Optional)

If I skip classes and I pass these classes, then I read the text books.
I do not read the text books.
I pass these classes
$\therefore$ I am smart.

7. (Optional) Consider the following premises.

   (i) It is not sunny this afternoon and it is colder than yesterday.

   (ii) We will go swimming only if it is sunny.

   (iii) If we do not go swimming, then we will take a canoe trip.

   (iv) If we take a canoe trip, then we will be home by sunset.

   From the above premises (i)-(iv), does the conclusion that *we go home by sunset* make a valid argument? Explain your answer by using rules of inferences.

8. (Optional) The famous detective Percule Hoirot was called in to solve a baffling murder mystery. He determined the following facts:

   (a) Lord Hazelton, the murdered man, was killed by a blow on the head with a brass candlestick.

   (b) Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder.

   (c) If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine.

   (d) If Lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.

   (e) If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.

   (f) If Sara was in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.

   Is it possible for the detective to deduce the identity of the murderer from these facts? If so, who did murder Lord Hazelton? (Assume there was only one cause of death.) Explain your answer by using rules of inferences.