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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

Table 1

Student	Y_i	X_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$x_i y_i$
1	2.8	63	-14.625	-0.4125	6.0328125
2	3.4	72	-5.625	0.1875	-1.0546875
3	3.0	78	0.375	-0.2125	-0.0796875
4	3.5	81	3.375	0.2875	0.9703125
5	3.6	87	9.375	0.3875	3.6328125
6	3.0	75	-2.625	-0.2125	0.5578125
7	2.7	75	-2.625	-0.5125	1.3453125
8	3.7	90	12.375	0.4875	6.0328125
Σ	25.7	621			$\Sigma = 17.4375$

$\bar{Y} = 3.2125$ $\quad \bar{X} = 77.625$ $\quad \Sigma x_i^2 = 511.875$

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} \qquad \hat{\beta}_2 = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$$

$$\hat{\beta}_1 = 3.2125 - 0.0340659(77.625) \qquad = \frac{17.4375}{511.875} = 0.0340659$$

$$= 0.568132$$

$$\hat{Y}_i = 0.568132 + 0.0340659(x_i)$$

With the exam score increase by 1 unit, on average, GPA will increase by 0.0340659 units. With no score on exam, student will get GPA equal to 0.568132.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

$\hat{Y}_1 = 2.714284$ $\quad \hat{Y}_2 = 3.020877$ $\quad \hat{Y}_3 = 3.225452$ $\quad \hat{Y}_4 = 3.32765$ $\quad \hat{Y}_5 = 3.532045$ $\quad \hat{Y}_6 = 3.123255$

$\hat{u}_1 = 0.085716$ $\quad \hat{u}_2 = 0.379123$ $\quad \hat{u}_3 = -0.225452$ $\quad \hat{u}_4 = 0.17235$ $\quad \hat{u}_5 = 0.067955$ $\quad \hat{u}_6 = -0.123255$

$\hat{Y}_7 = 3.123255$ $\quad \hat{Y}_8 = 3.634243$ $\quad \Sigma \hat{u}_i = 0.001061 \approx 0$

$\hat{u}_7 = -0.423255$ $\quad \hat{u}_8 = 0.065757$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

$$Var(\hat{u}_i) = \frac{\Sigma \hat{u}_i^2}{n-2} = \frac{0.4348931}{6} = 0.0724822 = \hat{\sigma}^2$$

$$Var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n \Sigma x_i^2} \hat{\sigma}^2 = \frac{48717}{8(511.875)} (0.0724822) = 0.862299$$

$$Var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\Sigma x_i^2} = \frac{0.0724822}{511.875} = 0.0001416$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

mean : 20 9.1

Σ = 200 91

$\Sigma = 394$

$\Sigma x_i^2 = (100+64+36+16+4) \times 2 = 440$

2. Data is listed in the table

Σx_1^2 :

X_i	Y_i	$(x_i-\bar{x})$	$(y_i-\bar{y})$	$x_i y_i$
10	0	-10	-9.1	91
12	2	-8	-7.1	56.8
14	5	-6	-4.1	24.6
16	6	-4	-3.1	12.4
18	7	-2	-2.1	4.2
22	10	2	0.9	1.8
24	10	4	0.9	3.6
26	15	6	5.9	35.4
28	16	8	6.9	55.2
30	20	10	10.9	109

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

2.1) $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$= 9.1 - (17.909091)$

$= -8.809091.$

$\hat{\beta}_2 = \dfrac{\Sigma x_i y_i}{\Sigma x_i^2}$

$= \dfrac{394}{440} = 0.895455$

$\hat{Y}_i = -8.8091 + 0.8955(X_i)$

2.2) $\hat{Y}_1 = 0.1459$ $\quad \hat{u}_1 = -0.1459$

$\hat{Y}_2 = 1.9369$ $\quad \hat{u}_2 = 0.0631$

$\hat{Y}_3 = 3.7279$ $\quad \hat{u}_3 = 1.2721$

$\hat{Y}_4 = 5.5189$ $\quad \hat{u}_4 = 0.4811$

$\hat{Y}_5 = 7.3099$ $\quad \hat{u}_5 = -0.3099$

$\hat{Y}_6 = 10.8919$ $\quad \hat{u}_6 = -0.8919$

$\hat{Y}_7 = 12.6829$ $\quad \hat{u}_7 = -2.6829$

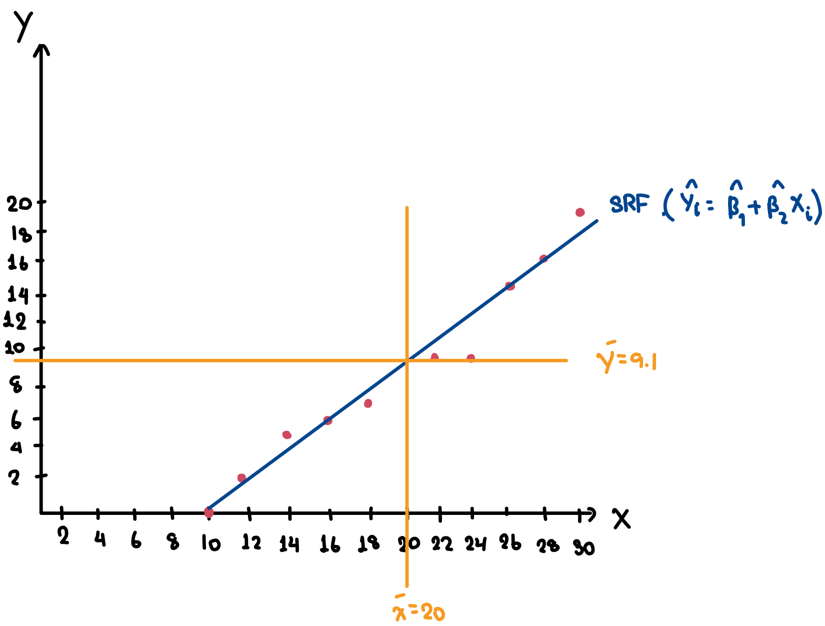
$\hat{Y}_8 = 14.4739$ $\quad \hat{u}_8 = 0.5261$

$\hat{Y}_9 = 16.2649$ $\quad \hat{u}_9 = -0.2649$

$\hat{Y}_{10} = 18.0559$ $\quad \hat{u}_{10} = 1.9441$

$\Sigma \hat{u}_i = -0.009 \approx 0$

2.3)



$\hat{Y}_i = -8.8091 + 0.8955(20) = 9.1009 \approx 9.1 = \bar{Y}$

2.4) $x_i = 18$, $\hat{Y}_i = 7.3099$ from 2.2

2.5) $Var(\hat{u}_i) = \sigma^2 = \dfrac{\Sigma \hat{u}_i^2}{n-2} = \dfrac{14.09092}{8} = 1.761365$

$Var(\hat{\beta}_2) = \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{1.761365}{440} = 0.004$

$Var(\hat{\beta}_1) = \dfrac{\Sigma X_i^2}{n} \cdot \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{444}{10}(0.004) = 1.776$

3. Consider the below regression function: consider the two-variable model

$Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$ → assumption 1, linear regression model.

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

As $\hat\beta_1$ is an estimator of β_1, $E(\hat\beta_1)$ should be equal to β_1

$\hat\beta_1 = \bar Y - \hat\beta_2 \bar X$, $\hat\beta_2 = \frac{\sum x_i Y_i}{\sum x_i^2}$ let $k = \frac{x_i}{\sum x_i^2}$

$\hat\beta_1 = \bar Y - \sum k Y_i \bar X$ and $Y_i = \beta_1 + \beta_2 X_i + u_i$

$$\hat\beta_1 = \frac{\sum Y_i}{n} - \sum k Y_i \bar X = \sum\left(\frac{1}{n} - k\bar X\right) Y_i$$

$$= \sum\left(\frac{1}{n} - k\bar X\right)(\beta_1 + \beta_2 X_i + u_i)$$

$$= \sum\left(\frac{\beta_1}{n} + \frac{\beta_2 X_i}{n} + \frac{u_i}{n} - k\bar X \beta_1 - k\bar X \beta_2 X_i - k\bar X u_i\right)$$

$$= \frac{\sum \beta_1}{n} + \beta_2 \frac{\sum X_i}{n} + \frac{\sum u_i}{n} - \bar X \beta_1 \overset{0}{\cancel{\sum k_i}} - \beta_2 \bar X \overset{1}{\cancel{\sum k_i X_i}} - \bar X \sum k_i u_i$$

$$\hat\beta_1 = \beta_1 + \beta_2 \bar X + \cancel{\frac{\sum u_i}{n}} - \beta_2 \bar X - \bar X \sum k_i u_i$$

take $E(\cdot)$ → $E(\hat\beta_1) = E(\beta_1) + \bar X\, E(\sum k_i u_i)$ → Assumption 3: Zero mean value of disturbance u_i

treat x as given $E(u_i | X_i)$

$E(\hat\beta_1) = E(\beta_1) + \bar X \sum k_i\, E(\overset{0}{\cancel{u_i}})$ as k is $f(x)$, k would be constant. $\sum k_i$ = constant

$$\boxed{E(\hat\beta_1) = \beta_1}$$

∴ $\hat\beta_1$ is an unbiased estimator of actual β_1