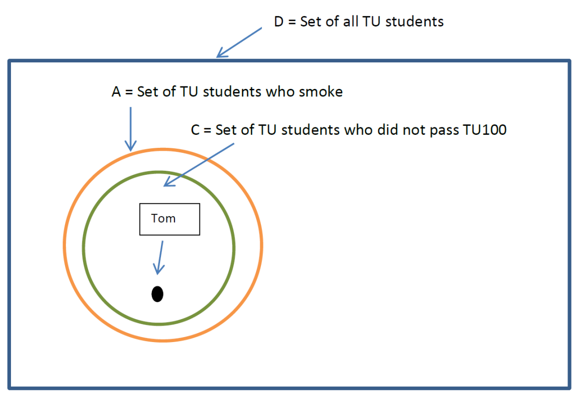
Solution: Quiz 2

1. Let the domain D be the set of all TU students. Suppose Tom $\in D$.
   Use the **diagram** to show that the following argument is valid or invalid.
   "None of the students who pass TU100 smoke ."
   "Tom does not pass TU100."
   $\therefore$ "Tom smokes."

   <u>Solution:</u>

   To use the diagram,
   let A be the set of TU students who smoke, and
   let C be the set of TU students who do not pass TU100.
   Then
   - the first premise "None of the students who pass TU100 smoke." tells us that "**All students who do not pass TU100 smoke**" (i.e. failing TU100 implies being a smoker), or "$C \subseteq A$."
   - the second premise "Tom does not pass TU100." tells us that "Tom is in the set C," or Tom $\in C$,
   - the conclusion tells us that "Tom is in the set A."



   From the diagram, since we know that "Tom" is an element in the set C which is inside the set A, then it is always true that "Tom" is in A. That is, the conclusion will always true. Therefore the argument is **valid**. $\blacksquare$

   To confirm this by using the universal rules of inferences.
   Let $P(x)$ be "x does not pass TU100,"
   $Q(x)$ be "x smokes."
   Then we can transform the given argument in the form of **universal modus ponens** as follows.
   For $x \in D$,

   $$\begin{array}{l} \forall x, P(x) \to Q(x) \\ P(a) \text{ for a particular } a = \text{ Tom} \\ \therefore Q(a) \end{array}$$

   That is, this argument is **valid** by universal modus ponens. $\blacksquare$

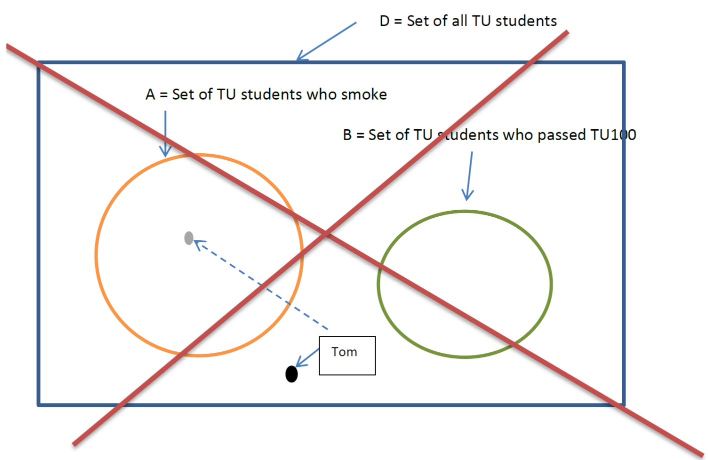
IMPORTANT REMARK ON DIAGRAM:

Suppose we
let A be be the set of TU students who smoke, and
let B be the set of TU students who pass TU100.

Let B^c be the complement of B, i.e. B^c is the set of elements of D that are not in B. Note that $B \cap B^c = \emptyset$.

- The first premise: "None of the students who pass TU100 smoke." tells us that "**All students who do not pass TU100 smoke**" (i.e. failing TU100 implies being a smoker), or "$B^c \subseteq A$." and so it is **not equivalent to** "$A \cap B = \emptyset$."
- The second premise implies "Tom"$\notin B$ or "Tom"$\in B^c$.
- The conclusion implies "Tom" $\in A$

Therefore, if we define the set A and B as above and use $A \cap B = \emptyset$ for the first premise, we will have the following diagram that **does not describe** the original argument. This diagram gives the answer "*invalid*" which is wrong.



2. Let $D = \{(-1,1),(0,0),(0,2),(1,2)\}$ be the domain of variable (x,y) for the predicate $P(x,y)$ defined as

$$P(x,y): \ \forall b \in \mathbb{Z}^+, \quad xy < b.$$

Determine the truth set of the predicate $P(x,y)$.

<u>**Solution:**</u> Let T_p be the truth set of P. Recall that each element of the truth set T_p is the order pair $(x,y) \in D$ that makes $P(x,y)$ true.

- For $(-1,1) \in D$, we have $xy = -1$ and $\quad P(-1,1): \ \forall b \in \mathbb{Z}^+, \quad -1 < b$
which is **true** because any $b \in \mathbb{Z}^+$ is positive and it is greater than -1. $\Rightarrow \quad (-1,1) \in T_q$

- For $(0,0) \in D$, we have $xy = 0$ and $\quad P(0,0): \ \forall b \in \mathbb{Z}^+, \quad 0 < b$
which is true because any $b \in \mathbb{Z}^+$ is positive and it has to be greater than 0. $\Rightarrow \quad (0,0) \in T_q$

- For $(0,2) \in D$, we have $xy = 0$ and $\quad P(0,2): \ \forall b \in \mathbb{Z}^+, \quad 0 < b$
which is true because any $b \in \mathbb{Z}^+$ is positive and it has to be greater than 0. $\Rightarrow \quad (0,2) \in T_q$

- For $(1,2) \in D$, we have $xy = 2$ and $\quad P(1,2): \ \forall b \in \mathbb{Z}^+, \quad 2 < b$
which is false because we can find $b \in \mathbb{Z}^+$, e.g. a counterexample $b = 1$ that is **not** greater than 2. $\Rightarrow \quad (1,2) \notin T_q$

From above, we have considered all elements (x,y) in D and $P(x,y)$ is true when $(x,y) = (-1,1),(0,0),(0,2)$. Therefore the truth set is $T_q = \{(-1,1),(0,0),(0,2)\}$. ■