Consumer's Problem. The consumer wants to maximize satisfaction (utility) by deciding what and how much to consume under the limitation of income.

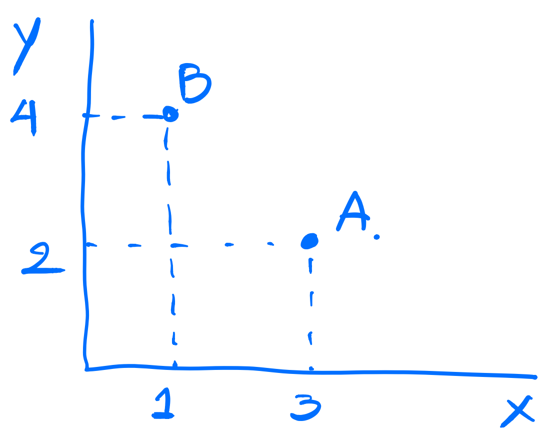
2 types of Utility.

1) **Cardinal Utility** - we can assign numerical value to the satisfaction of consumption

eg. a hamburger = 3 units of utility.

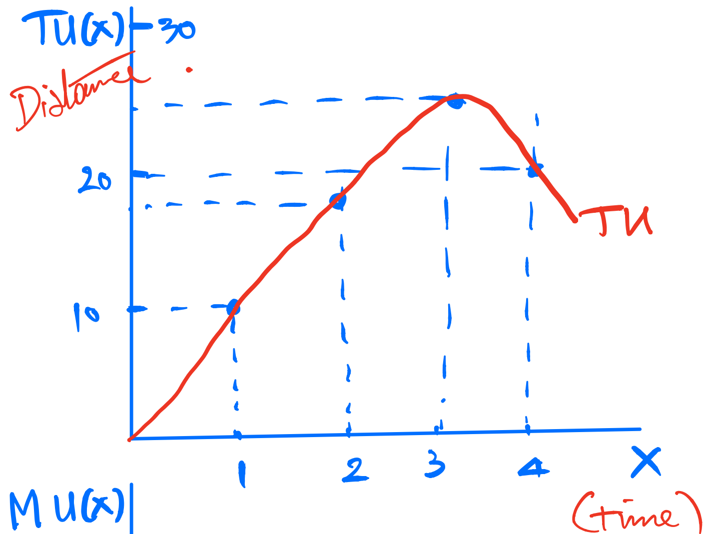
2) **Ordinal Utility** - consumer is able to say which bundle of goods he prefers to which bundle.

Def. A bundle of goods is a quantity of good X and a quantity of good Y.



Cardinal Utility

X	Total Utility (TU)	Marginal Utility (MU) Distance
0	0	
1	10	10
2	18	8
3	24	6
4	20	−4

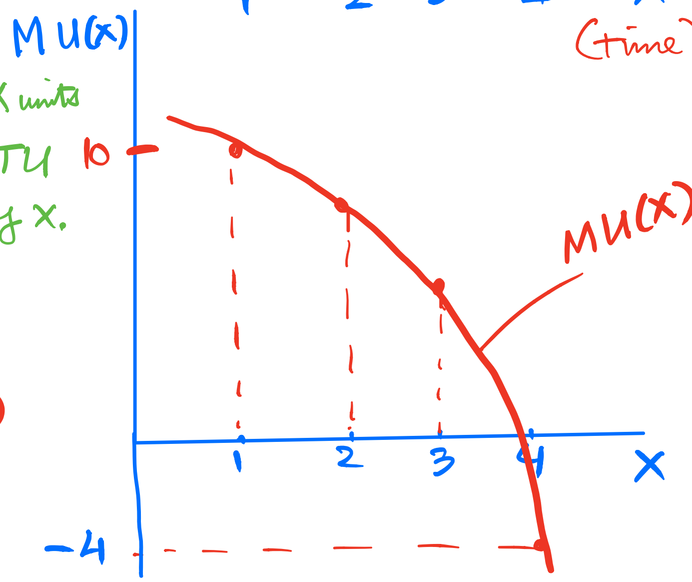


TU(X) = total Utility from consuming X units

MU(X) = Marginal Utility = change in TU from consuming 1 more unit of X.

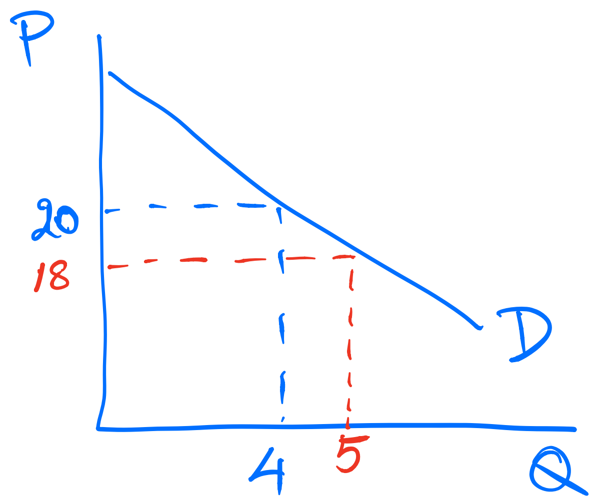
= rate of change of TU per unit change in X

$$MU(x) = \frac{d\,TU(x)}{dx}.$$



After certain amount of consumption of X, the MU of X always is decreasing.

– This is one explanation why the Demand has negative slope.



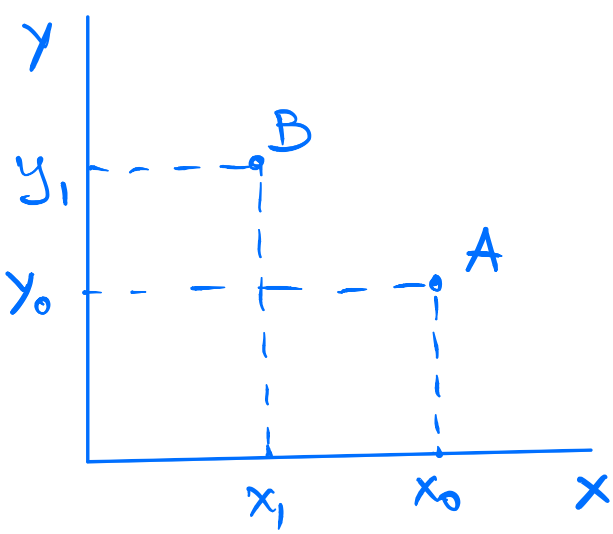
To make the buyer buy the 5^{th} unit, the price has to decrease because MU of the 5^{th} unit is lower than the 4^{th}.

<u>Ordinal Utility</u> - assume the consumer has 2 products to consume $(X+Y)$

A bundle is a point in X-Y plane

Bundle $A = (x_0, y_0)$

$B = (x_1, y_1)$



The consumer can say one of these.

$A \succ B$ (A is preferred to B)

or $B \succ A$

or $A \sim B$ (The consumer is indifferent between A + B)

$A \succsim B$ (A is at least as good as B)

or $B \succsim A$

or $A \sim B$.

Assumptions

The consumer is assumed to be rational.

1) Given any two bundles A + B, the consumer can say exactly one of these

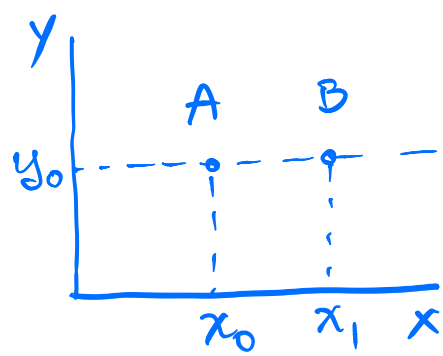
a) $A \succ B$
b) $B \succ A$
c) $A \sim B$

(or)

a) $A \succsim B$
b) $B \succsim A$
c) $A \sim B$

2) More is always better

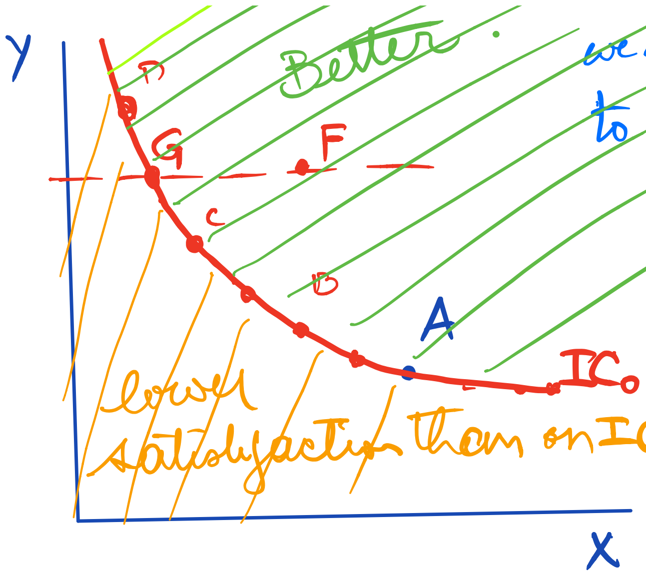
$B \succ A$ always!



3) Transitivity.

If $A \succ B$ $(\succsim)$, and $B \succ C$ $(\succsim)$, then $A \succ C$ $(\succsim)$.

4) Given any bundle A, the consumer can tell all other bundles that are equally preferred as A.



we can gather all these point to form a curve called **Indifference Curve (IC)**

= IC is a curve whose every point gives the same satisfaction to the consumer.

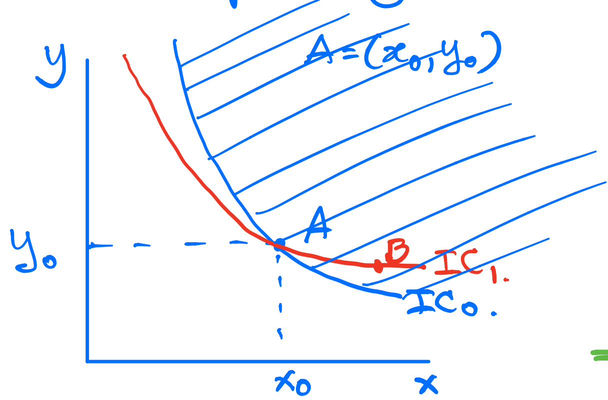
$F \succ G$ because more is always better

$G \sim A$ because G + A are on the same IC.

$\therefore F \succ A$ by Transitivity.

Properties of IC.

1). For any given bundle (x_0, y_0). there is exactly one IC passing through it.



For IC_0, we have the area that is better than IC_0

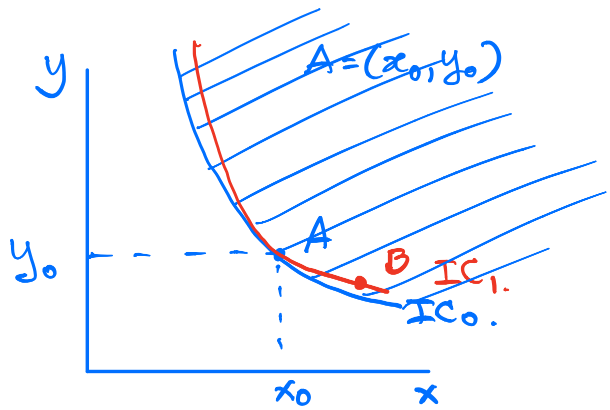
so $B \succ A$.

But according to IC_1

$B \sim A$

$\Rightarrow$ Inconsistency!

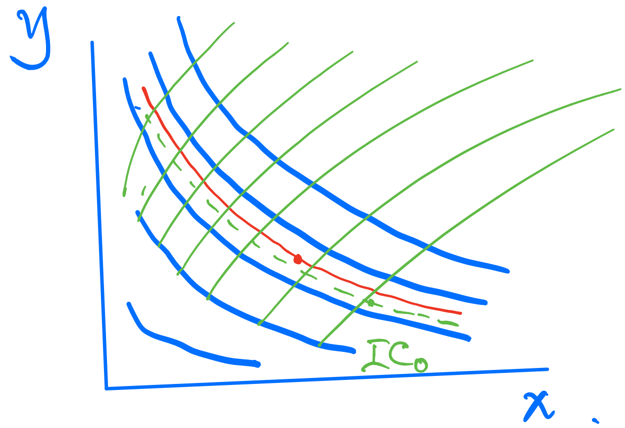
$\therefore$ IC's cannot intersect.



By same reasoning, IC_0 and IC_1 cannot be tangent at any point A.

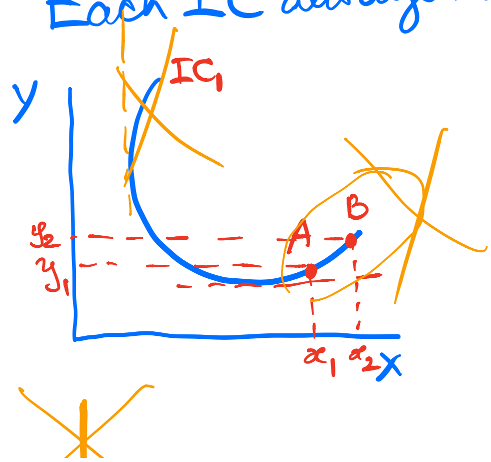
∴ IC's cannot be tangent to each other.

2) There are infinite number of IC's, each never intersect, or is tangent to another.



3) Higher IC means higher satisfaction.

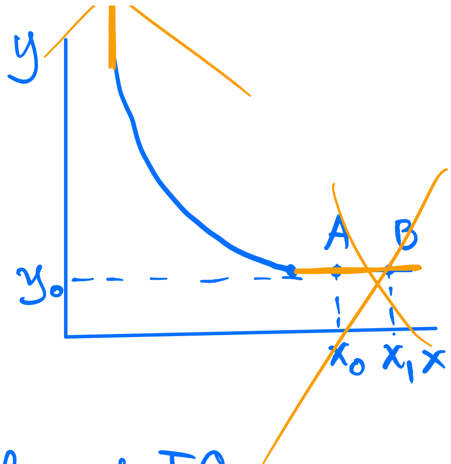
4). Each IC always has negative slope.

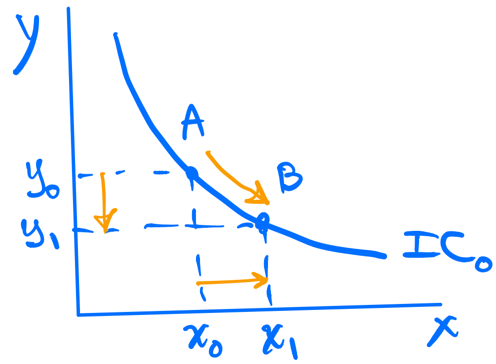


B ≻ A because we have more of X + Y at B.

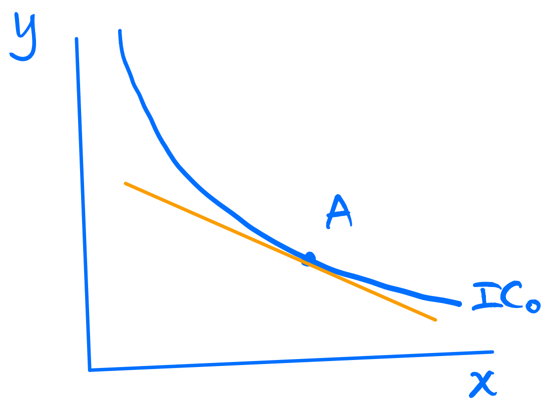
but B ~ A - according to IC_1

∴ Inconsistent.

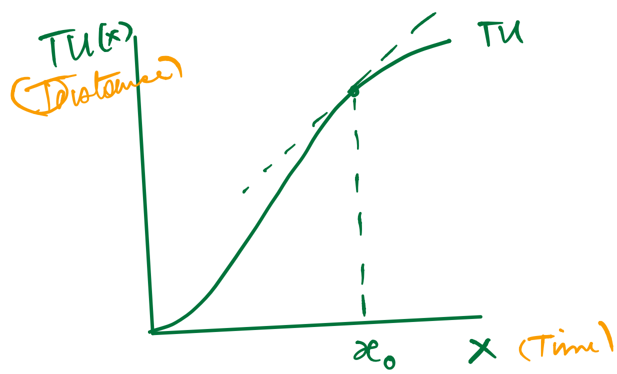




<u>Slope of IC</u> at a point on a given IC, the slope can be found by drawing a tangent line



Slope at A = slope of tangent line.



$$MU(x_0) = \frac{d\,TU(x_0)}{dx} = 3$$

∴ If we consume 1 more unit of x, we have ≈3 unit more of utility

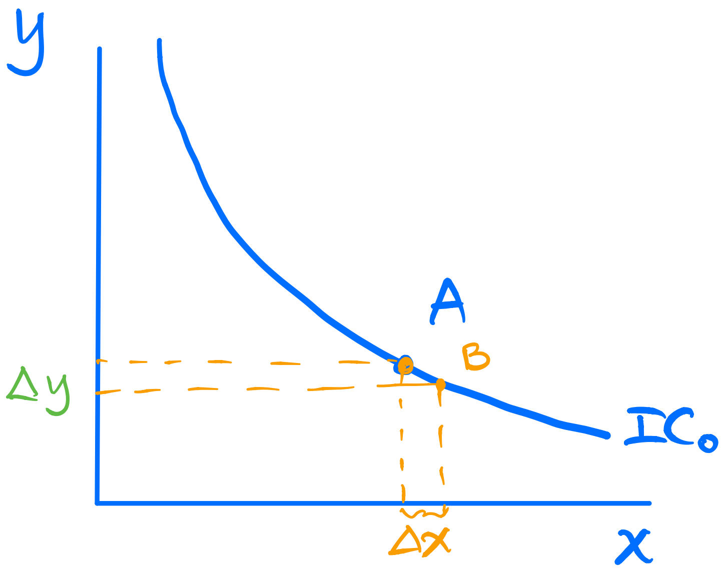
0.5	≈ 3(0.5)
0.1	≈ 3(.1)
Δx	$\approx 3\Delta x$.

$$\frac{d\,TU(x_0)}{dx} = MU(x_0)$$

$$\Delta TU(x_0) \approx MU(x_0)\cdot\Delta x.$$

$\Delta x \rightarrow dx$.

$d\,TU(x_0) = MU(x_0)\,dx$



A & B are on the same IC and are very close together.

From A to B, more x by Δx. $\Rightarrow$ $\Delta TU = MU_x \Delta x$ ①

less y by Δy $\Rightarrow$ $\Delta TU = MU_y \cdot \Delta y$. ②

Since A & B are on same IC_0

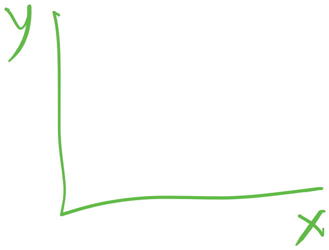
$$\underset{(+2.5)}{MU_x \Delta x} + \underset{(-2.5)}{MU_y \Delta y} = 0.$$

$$MU_y \Delta y = -MU_x \Delta x$$

$$\Delta y = -\frac{MU_x}{MU_y} \Delta x.$$

Slope of IC at A. $= \dfrac{\Delta y}{\Delta x} = \dfrac{-MU_x}{MU_y} = \dfrac{dy}{dx}$.

(if we let $\Delta x \to dx$, $\Delta y \to dy$.) = ratio of MU of X over MU of Y.

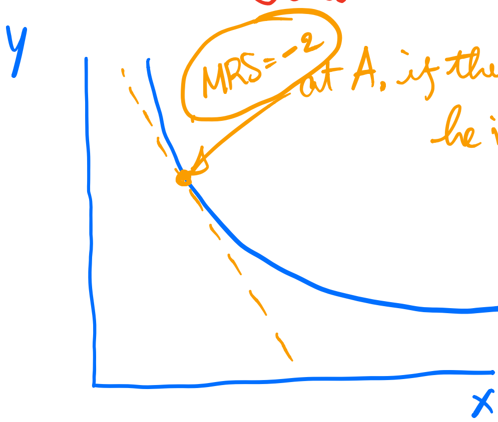


= Marginal rate of Substitution.

If $MU_x = \cancel{5}\ 10$, $MU_y = 5$.

$\Rightarrow$ slope $= \cancel{-1}\ -2$

slope of IC = MRS.
= the exchange rate between X + Y in the mind of the consumer.

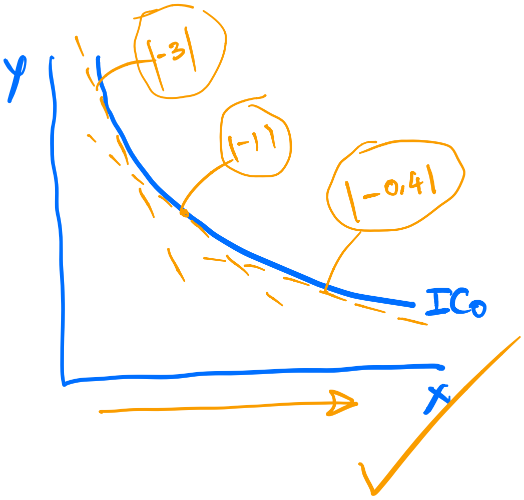


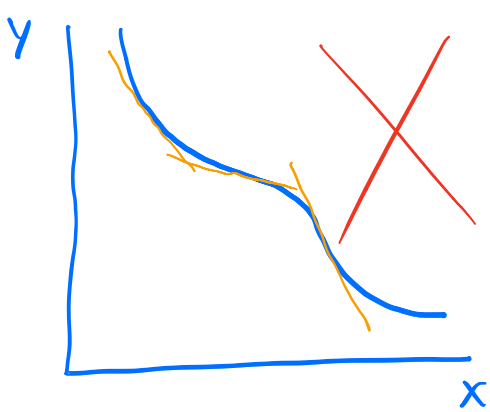
at A, if the consumer has 1 more unit of X he is willing to sacrifice 2 units of Y

+
$MU_x = 10$

$MU_y = 5$
$2 \times MU_y = 10$
−

5 As we increase X, |MRS| is assumed to be decreasing.





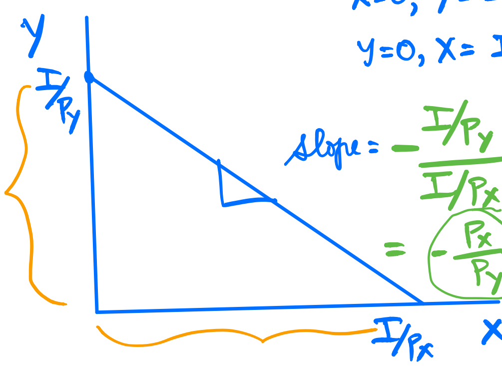
Budget Line. The consumer is assumed to have a fixed income that he can buy X at fixed price P_x and Y ——— " P_y.

Consumption Problem
— max satisfaction under Budget (income) limitation

The equation of budget line can be written as .

$$P_x X + P_y Y = I.$$

$X=0,\ Y = I/P_y$
$Y=0,\ X = I/P_x$

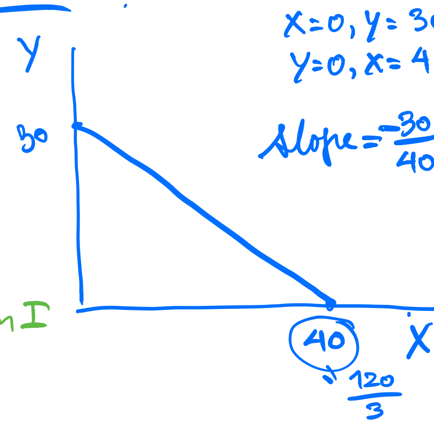


$$\text{slope} = -\frac{I/P_y}{I/P_x} = -\frac{P_x}{P_y}$$

does not depend on I

Ex. $3X + 4Y = 120.$

$X=0,\ Y=30$
$Y=0,\ X=40$



$$\text{Slope} = \frac{-30}{40} = \frac{-3}{4}.$$

Slope of Budget Line $= -\dfrac{P_x}{P_y}$ = − relative price of X over Y.

= exchange rate between X & Y the consumer can exchange X & Y in the market.

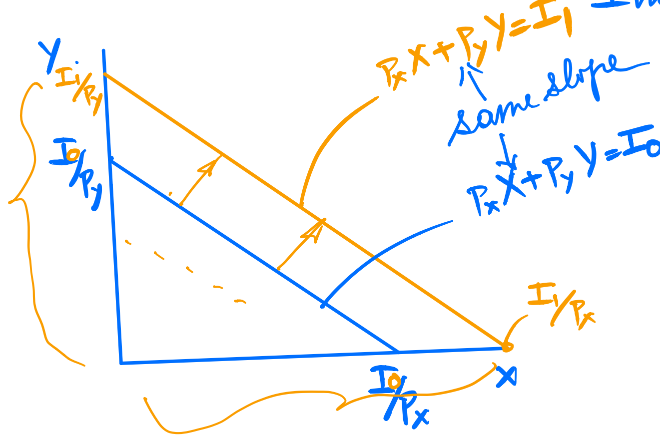
Ex If $P_x = 3,\ P_y = 3. \Rightarrow -\dfrac{P_x}{P_y} = -1$

$P_x = \underline{6},\ P_y = 3 \Rightarrow -\dfrac{P_x}{P_y} = -2.$

2 Possible changes of Budget line.

1) Income increases. (decreases)

Income increases from I_0 to I_1

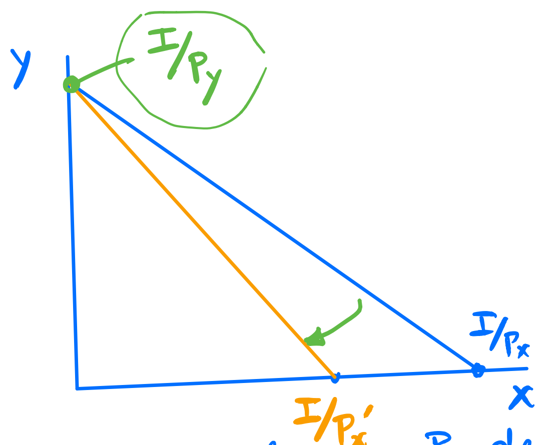


- higher Budget line means higher income (assuming fixed P_x & P_y).

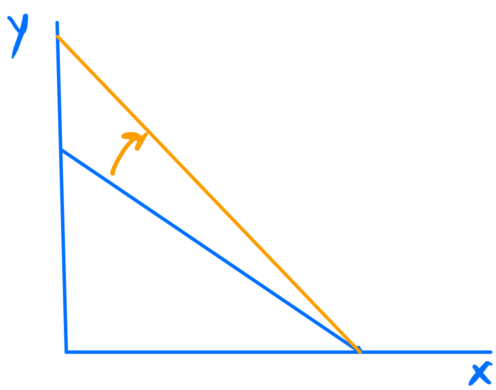
2) P_x increases (P_x decreases)

P_x increases to P_x'

$\Rightarrow \left|\frac{P_x'}{P_y}\right|$ is bigger.

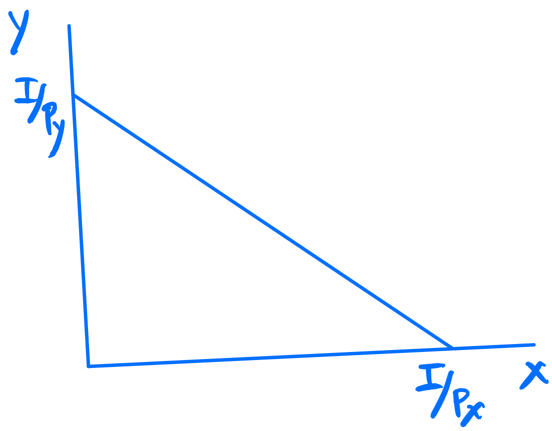


We can see that if P_y decreases, we have



- In economics, we always change 1 thing at a time.
  - no change in $P_x + P_y$ at the same time
    or P_x and I

But if the price $P_x + P_y$ change at the same time and at the same rate (i.e. increases by 10%)
(inflation)



H.W. Draw the new Budget line when

$P_x \to P_x' = 1.1\, P_x$

$P_y \to P_y' = 1.1\, P_y$.