CHAPTER 6: RISK AVERSION AND CAPITAL ALLOCATION TO RISKY ASSETS

PROBLEM SETS

13. Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

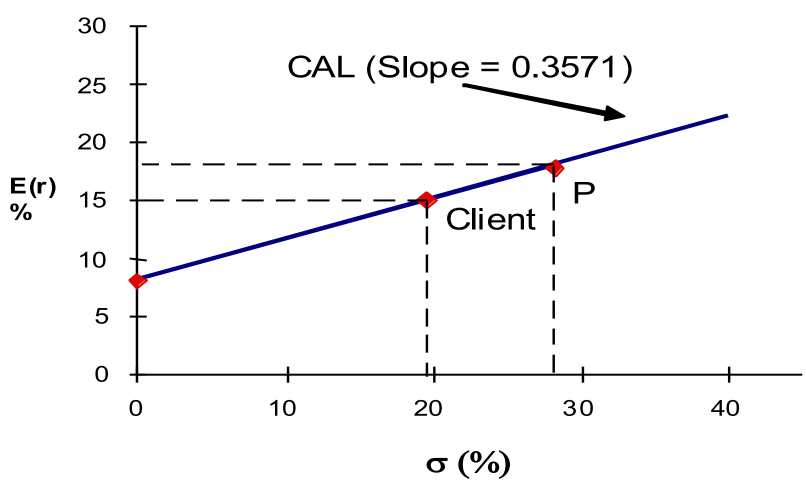
14. Investment proportions:

		30.0% in T-bills
	0.7 × 25% =	17.5% in Stock A
	0.7 × 32% =	22.4% in Stock B
	0.7 × 43% =	30.1% in Stock C

15. Your reward-to-volatility ratio: $S = \frac{.18 - .08}{.28} = 0.3571$

Client's reward-to-volatility ratio: $S = \frac{.15 - .08}{.196} = 0.3571$

16.



17. a. $E(r_C) = r_f + y \times [E(r_P) - r_f] = 8 + y \times (18 - 8)$

If the expected return for the portfolio is 16%, then:

$$16\% = 8\% + 10\% \times y \Rightarrow y = \frac{.16 - .08}{.10} = 0.8$$

Therefore, in order to have a portfolio with expected rate of return equal to 16%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

Client's investment proportions:		20.0% in T-bills
	$0.8 \times 25\%$ =	20.0% in Stock A
	$0.8 \times 32\%$ =	25.6% in Stock B
	$0.8 \times 43\%$ =	34.4% in Stock C

c. $\sigma_C = 0.8 \times \sigma_P = 0.8 \times 28\% = 22.4\%$

18. a. $\sigma_C = y \times 28\%$

If your client prefers a standard deviation of at most 18%, then:

$y = 18/28 = 0.6429 = 64.29\%$ invested in the risky portfolio

b. $E(r_C) = .08 + .1 \times y = .08 + (0.6429 \times .1) = 14.429\%$

19. a. $$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.18 - 0.08}{3.5 \times 0.28^2} = \frac{0.10}{0.2744} = 0.3644$$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 8 + 10 \times y^* = 8 + (0.3644 \times 10) = 11.644\%$

$\sigma_C = 0.3644 \times 28 = 10.203\%$

CHAPTER 7: OPTIMAL RISKY PORTFOLIOS

PROBLEM SETS

4. The parameters of the opportunity set are:

$E(r_S) = 20\%$, $E(r_B) = 12\%$, $\sigma_S = 30\%$, $\sigma_B = 15\%$, $\rho = 0.10$

From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \rho \times \sigma_S \times \sigma_B$]:

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

The minimum-variance portfolio is computed as follows:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$w_{Min}(B) = 1 - 0.1739 = 0.8261$

The minimum variance portfolio mean and standard deviation are:

$E(r_{Min}) = (0.1739 \times .20) + (0.8261 \times .12) = .1339 = 13.39\%$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{1/2}$$

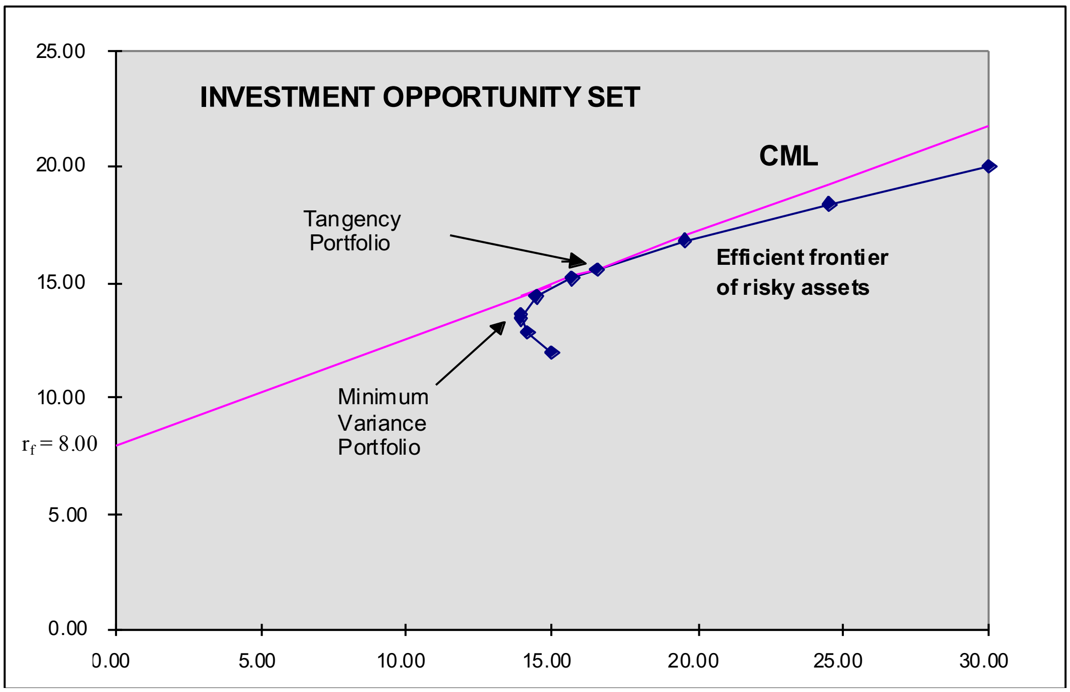
$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$= 13.92\%$

5.

Proportion in stock fund	Proportion in bond fund	Expected return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39%	82.61%	13.39%	13.92%	minimum variance
20.00%	80.00%	13.60%	13.94%	
40.00%	60.00%	15.20%	15.70%	
45.16%	54.84%	15.61%	16.54%	tangency portfolio
60.00%	40.00%	16.80%	19.53%	
80.00%	20.00%	18.40%	24.48%	
100.00%	0.00%	20.00%	30.00%	

Graph shown below.



6. The above graph indicates that the optimal portfolio is the tangency portfolio with expected return approximately 15.6% and standard deviation approximately 16.5%.

7. The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{[E(r_S)-r_f]\times\sigma_B^2 - [E(r_B)-r_f]\times Cov(r_S,r_B)}{[E(r_S)-r_f]\times\sigma_B^2 + [E(r_B)-r_f]\times\sigma_S^2 - [E(r_S)-r_f+E(r_B)-r_f]\times Cov(r_S,r_B)}$$

$$= \frac{[(.20-.08)\times 225]-[(.12-.08)\times 45]}{[(.20-.08)\times 225]+[(.12-.08)\times 900]-[(.20-.08+.12-.08)\times 45]} = 0.4516$$

$$w_B = 1-0.4516 = 0.5484$$

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P)$ = (0.4516 × .20) + (0.5484 × .12) = .1561

= 15.61%

σ_P = $[(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2}$

= 16.54%

8. The reward-to-volatility ratio of the optimal CAL is:

$$\frac{E(r_p)-r_f}{\sigma_p} = \frac{.1561-.08}{.1654} = 0.4601$$ .4601 should be .4603 (rounding)

9. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

$$E(r_C) = r_f + \frac{E(r_p)-r_f}{\sigma_P}\sigma_C = .08 + 0.4601\sigma_C$$ .4601 should be .4603 (rounding)

If $E(r_C)$ is equal to 14%, then the standard deviation of the portfolio is 13.03%.

b. To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1-y)\times r_f + y\times E(r_P) = r_f + y\times[E(r_P)-r_f] = .08 + y\times(.1561-.08)$$

Setting $E(r_C)$ = 14% we find: y = 0.7881 and (1 − y) = 0.2119 (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7881 × 0.4516 = 0.3559

Proportion of bonds in complete portfolio = 0.7881 × 0.5484 = 0.4322

10. Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (w_S) and the appropriate proportion in the bond fund ($w_B = 1 - w_S$) as follows:

$$.14 = .20 \times w_S + .12 \times (1 - w_S) = .12 + .08 \times w_S \Rightarrow w_S = 0.25$$

So the proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

$$\sigma_P = [(0.25^2 \times 900) + (0.75^2 \times 225) + (2 \times 0.25 \times 0.75 \times 45)]^{1/2} = 14.13\%$$

This is considerably greater than the standard deviation of 13.04% achieved using T-bills and the optimal portfolio.