Mainstream: Expected Utility

A&D Section 1.3

Expected Utility Theory

- **Expected Utility Theory** was developed by John Von Neumann and Oskar Morgenstern in an attempt to define rational behavior when people face *uncertainty*.
- However, expected utility theory is really set up to deal with *risk*, not *uncertainty*.
- A *risky* situation is one in which you know what the outcomes could be and can assign a probability to each outcome.
- *Uncertainty* is when you cannot assign probabilities or even cannot produce a list of possible outcomes.
- **Note** that in this lecture we will focus on the risk-averse people or investors only.

Prospect

- For almost all purposes, when considering decision-making under risk, it is sufficient to think in terms of wealth (monetary value).
- For simplicity, assume that there are only *two* states of the world: *low* wealth and *high* wealth.
- A **prospect** is a series of wealth outcomes, each of which is associated with a probability.
- Let $P(p_1, w_1, p_2, w_2)$ represent a prospect that outcome w_i occurs with probability p_i, $i = 1,2$.

Prospect

- For example, consider two risky outcomes in the below table.
- We can write $P_A(0.5, 5{,}0.5{,}25)$ to represent prospect A, where income of 5,000 baht occurs with prob. 50 percent and income of 25,000 baht with prob. 50 percent.
- Also, $P_B(0.5, 10{,}0.5{,}20)$ can be used to represent prospect B in the table.

(unit of income: thousand baht)	Low		High	
	Income	Prob.	Income	Prob.
A: Agent	5	0.5	25	0.5
B: Employee	10	0.5	20	0.5

Expected Utility Theory

- Under a set of assumptions, the probability-weighted expected value of the utility levels (expected utility) can be used as a basis for making decision under risk.
- For prospect $P(p_1, w_1, p_2, w_2)$, we can write the expected utility as
$$U(P) = p_1 \cdot u(w_1) + p_2 \cdot u(w_2)$$
where $u(x)$ represents the utility level of a certain outcome x.
- Function $u(\cdot)$ can be called ***Bernoulli utility function***.

Expected Utility Theory

- Consider the prospects

  $P_A(0.5, 5, 0.5, 25)$ and $P_B(0.5, 10, 0.5, 20)$,

  assume that utility a person is as shown in the following table.
- We can write

$$U(P_A) = 0.5(1.6) + 0.5(3.2) = 2.4$$
$$U(P_B) = 0.5(2.3) + 0.5(3.0) = 2.65$$

- $U(P_B) > U(P_A)$ the person should choose P_B.

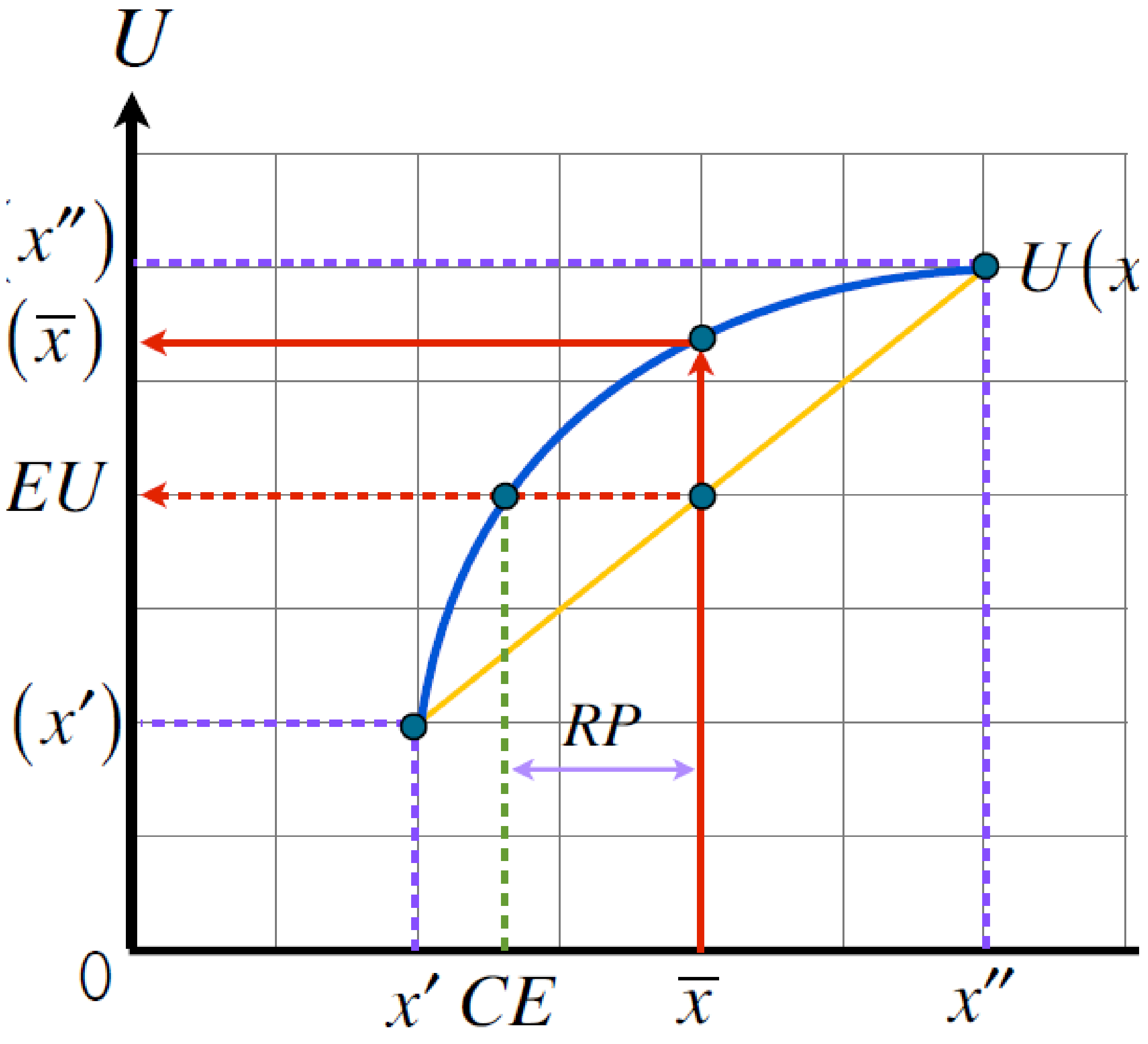
Income	utility
5	1.6
10	2.3
15	2.7
20	3.0
25	3.2

Expected Utility Theory

- Note that the argument in the Bernoulli utility function is the resulting *net wealth* of the person.
- Note also that in the literature, economists usually use functional form of the Bernoulli utility function as
$$u(x) = \frac{x^{1-A}}{1-A}, 0 < A < \infty$$
- Recall that a measures the curvature of the utility function (Arrow-Pratt coeff. of relative risk aversion) is
$$R(x) = -\frac{x \cdot u''(x)}{u'(x)}$$
- From the above function,
$$R(x) = -\frac{x(-Ax^{-A-1})}{x^{-A}} = A$$
- The higher the value of A, the more degree of risk aversion.
- ***See the Excel file

Certainty Equivalent

- A risk-averse person is willing to sacrifice some value to avoid risky situations.
- The **certainty equivalent** is defined as the wealth level that leads the decision-maker to be indifferent between a particular prospect and a certain wealth level.
- As an example, CE in the right-hand side figure is the certainty equivalent of prospect $P(0.5, x', 0.5, x'')$.
- **Risk premium** can be calculated from $\bar{x} - CE = RP$ in this figure.



Question 1			
A		A*	
Asset val.	Prob.	Asset val.	Prob.
30,000,000	100%	0	1%
		30,000,000	89%
		150,000,000	10%

Question 2			
B		B*	
Asset val.	Prob.	Asset val.	Prob.
0	89%	0	90%
30,000,000	11%	150,000,000	10%

Allias Paradox

Allias Paradox

- From many experiments in the past, a large number of people choose A over A* in question 1 and choose B* over B in question 2.
- But if the expected utility theory is correct,
  - In question 1, $U(A) > U(A^*)$ implies

$$U(A) = u(30{,}000{,}000) > 0.89u(30{,}000{,}000) + 0.1u(150{,}000{,}000) = U(A^*)$$

or

$$0.11u(30{,}000{,}000) > 0.1u(150{,}000{,}000)$$

  - In question 2, $U(B^*) > U(B)$ implies

$$0.1u(150{,}000{,}000) > 0.11u(30{,}000{,}000)$$

- Such choices are clearly contradictory.

Question 1			
A		A*	
Asset val.	Prob.	Asset val.	Prob.
30,000,000	89%	30,000,000	89%
30,000,000	11%	0	1%
		150,000,000	10%

Question 2			
B		B*	
Asset val.	Prob.	Asset val.	Prob.
0	89%	0	89%
30,000,000	11%	0	1%
		150,000,000	10%

Now, reconsider the prospect choices.

Framing

- Decision problems can be presented in many different ways, and some evidence suggests that people's decision are not the same across various presentations.
- A decision **frame** is defined to be a decision-maker's view of the problem and possible outcomes.
- A frame is affected by the presentation mode and the individual's perception of the question, as well as personal characteristics.
- This is a violation of expected utility theory, which rests on the assumption that people should have consistent choices, regardless of presentation.