Solution to Question Book for

Time Value of Money (Lecture Note 4)

ST-1

a. 

\$1,000 is being compounded for 3 years, so your balance at Year 4 is \$1,259.71:

$$FV_N = PV(1 + I)^N = \$1{,}000(1 + 0.08)^3 = \$1{,}259.71$$

Alternatively, using a financial calculator, input N = 3, I/YR = 8, PV =–1000, and PMT = 0; then solve for FV = \$1,259.71.

b. 

There are 12 compounding periods from Quarter 4 to Quarter 16.

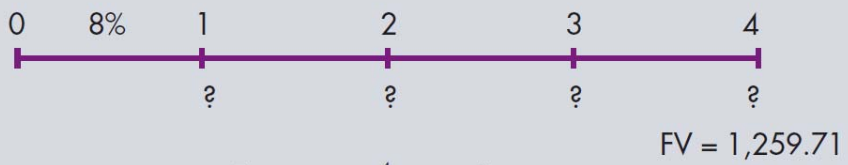
$$FV_N = PV\left(1 + \frac{I_{NOM}}{M}\right)^{NM} = FV_{12} = \$1{,}000(1.02)^{12} = \$1{,}268.24$$

Alternatively, using a financial calculator, input N = 12, I/YR = 2, PV = –1000, and PMT = 0; then solve for FV = \$1,268.24.

c. 

$$FVA_4 = \$250\left[\frac{(1 + 0.08)^4}{0.08} - \frac{1}{0.08}\right] = \$1{,}126.53$$

Using a financial calculator, input N = 4, I/YR = 8, PV = 0, and PMT = –250; then solve for FV = \$1,126.53.

d. 

$$PMT\left[\frac{(1 + 0.08)^4}{0.08} - \frac{1}{0.08}\right] = \$1{,}259.71$$

$$PMT(4.5061) = \$1{,}259.71$$

$$PMT = \$279.56$$

Using a financial calculator, input N = 4, I/YR = 8, PV = 0, and FV = 1259.71; then solve for PMT = –\$279.56.

ST-2 a. Set up a time line like the one in the preceding problem:

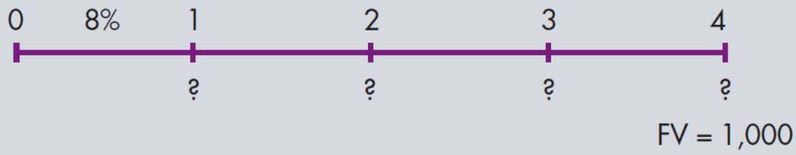
Note that your deposit will grow for 3 years at 8%. The deposit at Year 1 is the PV, and the FV is $1,000. Here is the solution:

$$N = 3, \quad I/YR = 8, \quad PMT = 0, \quad FV = 1000; \quad \text{then } PV = \$793.83.$$

Alternatively,

$$PV = \frac{FV_N}{(1+I)^N} = \frac{\$1{,}000}{(1+0.08)^3} = \$793.83$$

b.



Here we are dealing with a 4-year annuity whose first payment occurs 1 year from today and whose future value must equal $1,000. Here is the solution: N = 4; I/YR = 8; PV = 0; FV = 1000; then PMT = $221.92. Alternatively,

$$PMT\left[\frac{(1+0.08)^4}{0.08} - \frac{1}{0.08}\right] = \$1{,}000$$

$$PMT(4.5061) = \$1{,}000$$

$$PMT = \$222.92$$

c. This problem can be approached in several ways. Perhaps the simplest is to ask this question: “If I received $750 1 year from now and deposited it to earn 8%, would I have the required $1,000 4 years from now?” The answer is “no”:

$$FV_3 = \$750(1.08)(1.08)(1.08) = \$944.78$$

This indicates that you should let your father make the payments rather than accept the lump sum of $750.

You could also compare the \$750 with the PV of the payments:

N = 4, I/YR = 8, PMT = −221.92, FV = 0; then PV = \$735.03.

Alternatively,

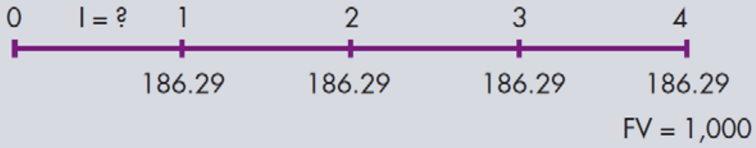
$$PVA_4 = \$221.92\left[\frac{1}{0.08} - \frac{1}{(0.08)(1+0.08)^4}\right] = \$735.03$$

This is less than the \$750 lump sum offer, so your initial reaction might be to accept the lump sum of \$750. However, it would be a mistake to do so. The problem is that, when you found the \$735.03 PVof the annuity, you were finding the value of the annuity *today*. You were comparing \$735.03 today with the lump sum of \$750 in 1 year. This is, of course, invalid. What you should have done was take the \$735.03, recognize that this is the PV of an annuity as of today, multiply \$735.03 by 1.08 to get \$793.83, and compare this \$793.83 with the lump sum of \$750. You would then take your father's offer to make the payments rather than take the lump sum 1 year from now.

d.

N = 3, PV = −750, PMT = 0, FV = 1000; then I/YR = 10.0642%.

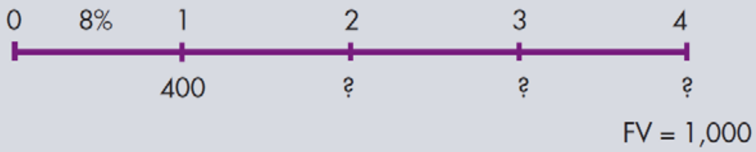
e.



N = 4, PV = 0, PMT = −186.29, FV = 1000; then I/YR = 19.9997%.

You might be able to find a borrower willing to offer you a 20% interest rate, but there would be some risk involved—he or she might not actually pay you your \$1,000!

f.



Find the future value of the original \$400 deposit:

$$FV_6 = PV(1+I)^6 = 400(1+0.04)^6 = \$400(1.2653) = \$506.12.$$

This means that, at Year 4, you need an additional sum of \$493.88: \$1,000.00 – \$506.12 = \$493.88. This amount will be accumulated by making 6 equal payments that earn 8% compounded semiannually, or 4% each 6 months: N = 6, I/YR = 4, PV = 0, FV = 493.88; then PMT = \$74.46. Alternatively,

$$\text{PMT}\left[\frac{(1+0.04)^6}{0.04}-\frac{1}{0.04}\right] = \$493.88$$

$$\text{PMT}(6.6330) = \$493.88$$

$$\text{PMT} = \$74.46$$

g.

$$\text{EFF\%} = \left(1+\frac{I_{NOM}}{M}\right)^M - 1.0$$

$$= \left(1+\frac{0.08}{2}\right)^2 - 1.0$$

$$= 1.0816 - 1 = 0.0816 = 8.16\%$$

ST-3 Bank A's effective annual rate is 8.24%:

$$\text{EFF\%} = \left(1+\frac{0.08}{4}\right)^4 - 1.0$$

$$= 1.0824 - 1 = 0.0824 = 8.24\%$$

Now Bank B must have the same effective annual rate:

$$\left(1+\frac{I}{12}\right)^{12} - 1.0 = 0.0824$$

$$\left(1+\frac{I}{12}\right)^{12} = 1.0824$$

$$1+\frac{I}{12} = (1.0824)^{1/12}$$

$$1+\frac{I}{12} = 1.00662$$

$$\frac{I}{12} = 0.00662$$

$$I = 0.07944 = 7.94\%$$

Thus, the two banks have different quoted rates—Bank A's quoted rate is 8%, whereas Bank B's quoted rate is 7.94%—yet both banks have the same effective annual rate of 8.24%. The difference in their quoted rates is due to the difference in compounding frequency.

4-1 FV_5 = \$16,105.10.

4-2 PV = \$1,292.10.

4-3 I/YR = 8.01%.

4-4 N = 11.01 years.

4-5 N = 11 years.

4-6 FVA_5 = \$1,725.22;

$FVA_{5\ Due}$ = \$1,845.99.

4-7 PV = \$923.98;

FV = \$1,466.24.

4-8 PMT = \$444.89;

EAR = 12.6825%.

4-9 a. \$530.

b. \$561.80.

c. \$471.70.

d. \$445.00.

4-10 a. \$895.42.

b. \$1,552.92.

c. \$279.20.

d. \$160.99.

4-11 a. N = 10.24 or around 10 years.

b. N = 7.27 or around 7 years.

c. N = 4.19 or around 4 years.

d. N = 1.00 or around 1 year.

4-12 a. \$6,374.97.

b. \$1,105.13.

c. \$2,000.00.

d. (1) \$7,012.46.

(2) \$1,160.38.

(3) \$2,000.00.

4-13 a. \$2,457.83.

b. \$865.90.

c. \$2,000.00.

d. (1) \$2,703.61.

(2) \$909.19.

(3) \$2,000.00.

4-14 a. PV_A = \$1,251.25.

PV_B = \$1,300.32.

b. PV_A = \$1,600.

PV_B = \$1,600.

4-15 a. 7%.

b. 7%.

c. 9%.

d. 15%.

4-16 a. \$881.17.

b. \$895.42.

c. \$903.06.

d. \$908.35.

4-17 a. \$279.20.

b. \$276.84.

c. \$443.72.

4-18 a. \$5,272.32.

b. \$5,374.07.

4-19 a. Universal, EAR = 7%;

Regional, EAR = 6.14%.

4-20 a. PMT = \$6,594.94;

Interest1 = \$2,500;

Interest2 = \$2,090.51.

b. \$13,189.87.

c. \$8,137.27.

4-21 a. I = 14.87% # 15%.

4-22 I = 7.18%.

4-23 I = 9%.

4-24 a. $33,872.11.

b. (1) $26,243.16.

(2) $0.

4-25 N = 14.77 or around 15 years.

4-26 6 years; $1,106.01.

4-27 (1) $1,428.57.

(2) $714.29.

4-28 $893.26.

4-29 $984.88.

4-30 57.18%.

4-31 a. $1,432.02.

b. $93.07.

4-32 I_{NOM} = 15.19%.

4-33 PMT = $36,949.61.

4-34 First PMT = $9,736.96.