

EE325 Section 1 HW 2 Due Thursday February 20<sup>th</sup> (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a.  $X_i$  is total microeconomics exam point (total points are 100) and  $Y_i$  is GPA of each student.

Table 1.a

Student	$Y_i$	$X_i$
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$\hat{y}_i$	$\hat{u}_i$
-14.625	-0.4125	6.0326	213.8906	2.7143	0.0857
-5.625	0.1875	-1.0547	31.6406	3.0209	0.3791
0.375	-0.2125	-0.0797	0.1406	3.2253	-0.2253
3.375	0.2875	0.9703	11.3906	3.3275	0.1725
9.375	0.3675	3.4326	87.8906	3.5319	0.0689
-2.625	-0.2125	0.5576	6.8906	3.1231	-0.1231
-2.625	-0.5125	1.3453	6.8906	3.1231	-0.4231
12.375	0.4675	5.7726	153.1406	3.6341	0.0659
		$\Sigma = 17.4374$	$\Sigma = 511.675$		

$\bar{y} = 3.2125$   $\bar{x} = 77.625$

1.1 Now consider the two-variable  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Use OLS to find the estimator of  $\beta_0$  and  $\beta_1$ . (Note: NIID = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{17.4374}{511.675} = 0.0341$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 3.2125 - (0.0341)77.625 = 0.5655$$

1.2 For each observation  $i$ , find  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$   
 $\hat{u}_i = y_i - \hat{y}_i$

Table 1.a

Student	$Y_i$	$X_i$
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$\hat{y}_i$	$\hat{u}_i$
2.7143	0.0857
3.0209	0.3791
3.2253	-0.2253
3.3275	0.1725
3.5319	0.0689
3.1231	-0.1231
3.1231	-0.4231
3.6341	0.0659
	$\Sigma \hat{u}_i = 0$

1.3 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

$$Var(\hat{u}_i) = \frac{0.4347}{6} = 0.0725$$

$$Var(\hat{\beta}_1) = \frac{6^2}{SSE} \cdot \frac{6^2}{\sum(x_i - \bar{x})^2} = \frac{0.0725}{511.875} = 0.0001$$

$$Var(\hat{\beta}_0) = \frac{\sum x_i^2 - n\bar{x}^2}{n \sum(x_i - \bar{x})^2} = \frac{46.717(0.0001)}{2(511.875)} = 0.0119$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\hat{u}_i = y_i - \hat{y}_i$$

$$\hat{y}_i = -8.81 + (0.8955)(x_i)$$

2. Data is listed in the table

$X_i$	$Y_i$	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$\hat{y}_i$	$\hat{u}_i$
10	0	-10	-9.1	91	100	0.145	-0.145
12	2	-8	-7.1	56.8	64	1.936	0.064
14	5	-6	-4.1	24.6	36	3.727	1.273
16	6	-4	-3.1	12.4	16	5.518	0.482
18	7	-2	-2.1	4.2	4	7.309	-0.309
22	10	2	0.9	1.8	4	10.891	-0.891
24	10	4	0.9	3.6	16	12.682	-2.682
26	15	6	5.9	35.4	36	14.473	0.527
28	16	8	6.9	55.2	64	16.264	0.264
30	20	10	10.9	109	100	18.055	1.945
				394	440		

2.1 From the simple regression model  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Find estimators of

$\beta_0$  and  $\beta_1$  from the OLS method and interpret the meaning.

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$\hat{\beta}_1 = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_0 = 9.1 - (0.8955)(20)$$

$$= -8.81$$

$$\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$$

$$\bar{y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$$

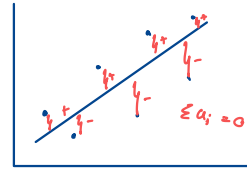
2.2 Find the value of  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\hat{u}_i = y_i - \hat{y}_i$$

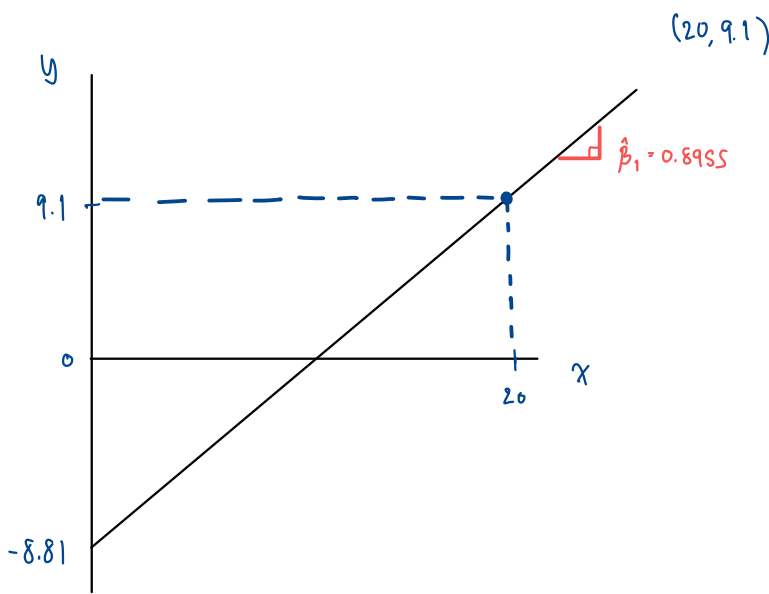
$x_i$	$y_i$	$\hat{y}_i$	$\hat{u}_i$
10	0	0.145	-0.145
12	2	1.936	0.064
14	5	3.727	1.273
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30	20	18.055	1.945

$$\sum_{i=0}^N (y_i - \hat{y}_i) = 0$$



$$\sum_{i=0}^N \hat{u}_i = 0$$

2.3 Plot graph and draw regression line. Does the line pass  $(\bar{X}, \bar{Y})$ ?



$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\hat{y} = -8.81 + 0.8955 x_i$$

(estimated  $y = \hat{y}$ )

2.4 If  $X_i = 16$ , what is the predicted Y?

$$x_i = 16 \quad \hat{y} = 5.518$$

2.5 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

$$var(\hat{u}_i) = \frac{\sum x_i^2}{n-2} = \frac{14.089}{18} = 0.7827$$

$$var(\hat{\beta}_0) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2} = 1.7271$$

$$var(\hat{\beta}_1) = 0.004$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where  $u_i \sim NID(0, \sigma^2)$ . Find an OLS estimator of  $\beta_1$ . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

Suppose  $x_i = x_i - \bar{x}$        $k_i = \frac{x_i}{\sum x_i^2}$

$$\begin{aligned} \hat{\beta}_1 &= \sum_{i=1}^n (y_i - \bar{y}) k_i \\ &= \sum_{i=1}^n (\beta_0 + \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x}) k_i \\ &= \sum_{i=1}^n \beta_1 (x_i - \bar{x}) k_i + \sum_{i=1}^n u_i k_i \\ &= \beta_1 \sum_{i=1}^n x_i \frac{x_i}{\sum x_i^2} + \sum_{i=1}^n u_i k_i \\ &= \beta_1 \frac{\sum x_i^2}{\sum x_i^2} + \sum_{i=1}^n u_i k_i \end{aligned}$$

$$E(\hat{\beta}_1) = E[\beta_1 + \sum_{i=1}^n u_i k_i]$$

SLR 4:  $E(u_i | x_i) = 0$

$$E(\hat{\beta}_1) = \beta_1 + \sum_{i=1}^n x_i E(u_i)$$

$$E(\hat{\beta}_1) = \beta_1$$