Matrix Algebra (Quick Review)

(For more basic background, please read Wooldridge Appendix D)

1 Matrix Operation

- Addition - $A+B$ is possible when A and B are of the same dimension.

$$\begin{aligned} A &= \begin{bmatrix} 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \end{bmatrix}_{2\times 4}, \quad B = \begin{bmatrix} 1 & 0 & -1 & 3 \\ -2 & 0 & 1 & 5 \end{bmatrix}_{2\times 4} \\ A+B &= \begin{bmatrix} 3 & 3 & 3 & 8 \\ 4 & 7 & 9 & 14 \end{bmatrix}_{2\times 4} \end{aligned}$$

- Subtraction

$$A-B = \begin{bmatrix} & & & \\ & & & \end{bmatrix}_{2\times 4}$$

- Scalar Multiplication

$$\lambda A = \lambda \left[a_{ij}\right]_{i\times j}$$

Suppose $\lambda = 2$

$$\begin{aligned} A &= \begin{bmatrix} -3 & 5 \\ 8 & 7 \end{bmatrix} \\ \lambda A &= \begin{bmatrix} & \\ & \end{bmatrix} \end{aligned}$$

- Matrix Multiplication

$A_{(M\times N)}B_{(O\times P)}$ is possible if an only if $N=O$ ($\leftarrow$ Letter O). The multiplication result would be of dimension ___________.

Let

$$\begin{aligned} A &= \begin{bmatrix} 3 & 4 & 7 \\ 5 & 6 & 1 \end{bmatrix}_{2\times 3}, \quad B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 6 & 2 \end{bmatrix}_{3\times 2} \\ C &= A\times B = \begin{bmatrix} & \\ & \end{bmatrix}_{2\times 2} \\ C &= \begin{bmatrix} & \\ & \end{bmatrix}_{2\times 2} \\ C_{ij} &= \end{aligned}$$

- Matrix Differentiation

Rule1 - If $a' = \begin{bmatrix} a_1 & a_2 & a_3 & ... & ... & a_n \end{bmatrix}$ and $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ ... \\ x_n \end{bmatrix}$

Then

- Rule2 - If $X'AX = \begin{bmatrix} x_1 & x_2 & x_3 & ... & ... & x_n \end{bmatrix}_{1\times n} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{n\times n} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ ... \\ x_n \end{bmatrix}_{n\times 1}$

Then

2 OLS Estimation using Matrix Notation

Let

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ .. \\ .. \\ .. \\ .. \\ \\ y_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} 1 & x_{11} & x_{21} & ... & x_{k1} \\ 1 & x_{12} & & & .. \\ 1 & x_{13} & & & .. \\ .. & .. & & & .. \\ \\ \\ \\ \\ 1 & x_{1n} & & & x_{kn} \end{bmatrix}_{n\times(k+1)} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \\ ... \\ \hat{\beta}_k \end{bmatrix}_{(k+1)\times 1} + \begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \\ .. \\ .. \\ .. \\ .. \\ .. \\ \hat{u}_n \end{bmatrix}_{n\times 1}$$

Example: Let the estimated model be

$$y = \beta_0 + \beta_1 x_1 + u$$

Let
$\hat{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{bmatrix}_{2\times 1}$

$X'X =$

3 Variance-Covariance Matrix of $(\hat{\beta})$

$$
\begin{aligned}
\text{From Variance-Covariance Matrix of } (\hat{\beta}) &= E\{[\hat{\beta} - E(\hat{\beta})][\hat{\beta} - E(\hat{\beta})]'\} \\
&= E\{[\hat{\beta} - \beta][\hat{\beta} - \beta]'\} \\
\text{from } \hat{\beta} &= (X'X)^{-1}X'Y
\end{aligned}
$$