Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c. **Please read and answer the following questions carefully and completely.**

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

$U'(C_0) = R_f \delta E[U'(C_1)]$

$\frac{1}{R_f} = \delta E\left[\frac{1/C_1}{1/C_0}\right]$

$= \delta E\left[C_0/C_1\right]$

$U'(C) = \frac{1}{C}$

if there is only risk ; $R_f = \frac{1}{\delta}\left(\frac{C_1}{C_0}\right)$

there is a positive relationship b/w risk free rate and consumption

when the interest is high , consumption growth is high

Score....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

from $R_f = \frac{1}{\delta}\left(\frac{C_1}{C_0}\right)$ F.O.C $\frac{\partial R_f}{\partial \frac{C_1}{C_0}} = \frac{1}{\delta}$

$\frac{\partial C_1/C_0}{\partial R_f} = \delta$

multiply by $\frac{C_1/C_0}{1/R_f}$ both sides; $\frac{\partial C_1/C_0 / \frac{C_1}{C_0}}{\partial R_f / R_f} = \frac{\delta / \frac{C_1}{C_0}}{1/R_f}$

$\frac{R_f}{\frac{C_1}{C_0}} \frac{\partial \frac{C_1}{C_0}}{\partial R_f} = \frac{\delta C_0 R_f}{C_1}$

substitute $R_f = \frac{1}{\delta}\left(\frac{C_1}{C_0}\right)$; $\frac{R_f}{\frac{C_1}{C_0}} \frac{\partial \frac{C_1}{C_0}}{\partial R_f} = \frac{\delta C_0}{C_1}\left[\frac{1}{\delta}\left(\frac{C_1}{C_0}\right)\right]$

$\therefore \epsilon = \frac{R_f}{\frac{C_1}{C_0}} \frac{\partial \frac{C_1}{C_0}}{\partial R_f} = 1$

There is one-for-one relationship b/w R_f and $\left(\frac{C_1}{C_0}\right)$. If next year the interest rate drop, consumption will decrease by the same proportion

A substitution effect : when the interest rate is high people will save more to enjoy higher future consumption

An income effect : when the interest rate is high people become relatively richer, so they will consume more both period

since $\epsilon = 1$, IE and SE exactly offset each other

Score...................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$$m_{01} = \frac{\delta U'(C_1)}{U'(C_0)} = \frac{\delta \, 1/C_1}{1/C_0} = \frac{\delta C_0}{C_1}$$

$$P_i = E[m_{01} X_i]$$

$$= E\left[\frac{\delta C_1}{C_0} X_i\right]$$

∴ An asset that pays good payoff in the state that economy is good (high C_1), marginal utility is low (low $U'(C_1)$). Thus, the price will be small, vice versa

The current price is an expected discounted value of its payoffs, where the discount factor, m_{01}, is a random number depending on level of future consumption

Score....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

$U(C) = \ln(C) \qquad m_{01} = \delta \frac{C_0}{C_1} = \delta e^{\ln\left(\frac{C_0}{C_1}\right)}$

$$\frac{\sigma_{m_{01}}}{E[m_{01}]} = \frac{\sqrt{\cancel{\delta^2} Var[e^{\ln\left(\frac{C_0}{C_1}\right)}]}}{\cancel{\delta} E[e^{\ln(C_0/C_1)}]}$$

$$= \sqrt{\frac{E[e^{2\ln(C_0/C_1)}] - E[e^{\ln(C_0/C_1)}]^2}{E[e^{\ln(C_0/C_1)}]^2}}$$

$$= \sqrt{\frac{E[e^{2\ln(C_0/C_1)}]}{E[e^{\ln(C_0/C_1)}]^2} - 1}$$

$$= \sqrt{\frac{e^{2\mu_c + \frac{1}{2}2^2\sigma_c^2}}{e^{2(\mu_c + \frac{1}{2}\sigma_c^2)}} - 1} = \sqrt{\frac{\cancel{e}^{\cancel{2\mu_c} + \cancel{2}\sigma_c^2}}{\cancel{e^{2\mu_c + \sigma_c^2}}} - 1} = \sqrt{e^{\sigma_c^2} - 1} = \sqrt{\cancel{1} + \sigma_c^2 - \cancel{1}}$$

$$= \pm \sigma_c$$

$$= \sigma_c$$

Any assets that have sharpe ratio lower than or equal to σ will have no equity premium puzzle, or the model makes sense (share ratio < H & J bound)

If there is an asset whose return is perfectly negatively correlated with the m_{01} then the bound σ_c holds with equity