Midterm MA217 2015 (2 hour exam)

1. (a) For a function $f(x,y) = 250\left(\frac{5x}{2+x} + \frac{2y}{5+y}\right) - x - y$,

   determine all possible critical point(s).

   (b) Select **ONLY** critical point(s) from (a) that are in the 1st quadrant (x and y are positive) and use the **second derivative test** to classify it/them. (8 marks)

Ans: (48, 45) is a relative maximum

2. Use **Extreme Value theorem** to optimise

   the function $f(x,y) = xy - \frac{1}{x} - \frac{1}{y}$

   subject to the constraint $x \le 2$; $y \ge -2$ and $x \ge y$. (12 marks)

Ans: $f(-2,-2) = 5$ absolute maximum and $f(2,-2) = -4$ absolute minimum

3. (a) Use **Lagrange Multiplier method** to optimise

   the function $f(x,y,z) = 4x + 2y - 4z$

   subject to the constraint $x^2 + y^2 + z^2 = 36$.

   (b) **Approximate the minimum value** if the function changes to

   $f(x,y,z) = 4.02x + 2.01y - 4.02z$ subject to the same constrain $x^2 + y^2 + z^2 = 36$. (10 marks)

Ans: (a) $f(4,2,-4) = 36$ absolute maximum and $f(-4,-2,4) = -36$ absolute minimum, (b) -36.18

4. Optimise the function $f(x,y) = 3x^2 + xy + 2y^2$

   subject to the constraint $x + y \le 200$ and $y \le 100$. (12 marks)

Ans: $f\left(-\frac{50}{3}, 100\right) = 19{,}166.7$ absolute maximum and $f(0,0) = 0$ absolute minimum

5. An equation of the least square linear-regression line has the slope equal to -2.5 for the following data set

x	1	2	3	4	5
y	35	31	26	24	y_5

   (a) It is noted that the y value for the last dataset, y_5, is missing. Determine the missing value of y_5.

   (b) Predict the value of y corresponding to $x = 6$. (8 marks)

Ans: (a) y_5=26, (b) y_6=20.9