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Group Homework 2

Semester 2/2022 EE320 Introductory mathematical economics

Due date: Feb 15th 2022 (before midnight / B.E. moodle).

Note: Late homework will not be accepted. Use the format of filename as required; this will cost you two points if you don't follow the instruction.

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1. Consider the ice cream market in Bangkok. In April, the ice cream market demand and supply curves are given by the following equations where Q is the quantity of ice cream units, T is the level of temperature in degree Celsius, and P is the price in dollars per unit of ice cream:

$$\text{Demand: } Q = 10000 + 400 \times (T - 30) - 10P$$
$$\text{Supply: } Q = 2000 + 20P$$

Suppose that T = 40 Celsius, find the equilibrium price and quantity of ice cream in April using the Inverse matrix method.

2. Consider a modified version of the IS-LM model where government spending is tied to the level of GDP (Y).

$C = C_0 + C_1Y_d - C_2r, \qquad 0 < C_1 < 1$

$I = I_0 + I_1Y - I_2r, \qquad 0 < I_1 < 1$

$G = G_0 - G_1Y, \qquad 0 < G_1 < 1$

$T = T_0$

$M^s = M_0$

$L^d = L_0 + L_1Y - L_2r$

where C is the private consumption, I is the private investment, G is the government spending, T is tax, M^s is the level of money supply, L^d is the level of real money demand, and r is the level of nominal interest rate. Suppose that price is fixed equal to 1. All the coefficients are *non-negative*. Additionally, we assume that $I_1 + C_1 - G_1 < 1$.

a. Discuss about the nature of government behavior. Does the assumption over the behavior of government make sense in practice?
b. What does C_2 represent? What does it imply about the behavior of private consumption? Does the assumption make sense?
c. Derive the IS equation. Interpret the meaning of the IS equation. Discuss about the key relation derived from IS equation.
d. Calculate the slope of IS curve. When is the IS curve flat? What does the flat IS curve imply about the sensitivity of real GDP to the interest rate?
e. Use the IS equation and calculate the tax multiplier. How does the tax multiplier depend on slope of IS curve?

f. Derive the LM equation. Interpret the meaning of the LM equation. Discuss about the key relation derived from LM equation.

g. Calculate the slope of LM curve. When is the LM curve flat? What does the flat LM curve imply about the sensitivity of interest rate to the real GDP?

h. Write both IS and LM equations in terms of the matrix representation.

i. Solve for the equilibrium GDP and interest rate (Y^*, r^*) using the Cramer's rule. (*Caution*: you will get **ZERO** if you don't use the Cramer's rule.)

j. Calculate the multiplier of G_0 and the multiplier of M_0 *on* both Y^* and r^*, respectively. Discuss whether the multiplier is bigger or smaller than the case that government spending is purely exogenous, i.e. $G_1 = 0$.

The government is seeking for some advices on fiscal and monetary policy implementation. The goal of the government is to (i) *increase the real GDP (Y) by $100*, while (ii) *keeping the current level of interest rate stayed the same*. (That is, the government was thinking that the country is running into an unemployment situation, but the level of interest rate is now optimal.) Following the storyline given here and all your work that you have done before, answer the next two questions.

k. Can the government successfully achieve both goals by simply relying on *a single type of policy implemented*? That is, to achieve the two goals, would it work to either change the *government expenditure* or *money supply*, but not both at the same time? If yes, *under which conditions*?

l. If the condition that you assumed in (k) does not hold, what would you recommend to the government so that both goals can be simultaneously achieved? (Hint: think about an appropriate mixture of the two policies.)

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Question 1:

Demand: $Q = 10000 + 400 \times (\overset{\nearrow T=40}{T} - 30) - 10P \Rightarrow Q = 10{,}000 + 4{,}000 - 10P$

Supply: $Q = 2000 + 20P$

Endo: Q ; P

Step 1: LHS vs. RHS

Demand $Q + 10P = 14{,}000$

Supply $Q - 20P = 2{,}000$

$$Ax = d$$

$$x = \begin{bmatrix} Q \\ P \end{bmatrix} \Rightarrow \underbrace{\begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix}}_{A} \begin{bmatrix} Q \\ P \end{bmatrix} = \begin{bmatrix} 14{,}000 \\ 2{,}000 \end{bmatrix}$$

$$\det(A) = \begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix} = -20 - 10 = -30$$

$$\text{Cof}(A) = \begin{bmatrix} \underset{}{\overset{-20}{C_{11}}} & \overset{-1}{C_{12}} \\ \underset{-10}{C_{21}} & \underset{1}{C_{22}} \end{bmatrix}$$

$C_{11} = (-1)^2 \cdot -20 = -20$

$C_{12} = (-1)^3 \cdot 1 = -1$

$C_{21} = (-1)^3 \cdot 10 = -10$

$C_{22} = (-1)^4 \cdot 1 = 1$

$$\text{Adj}(A) = [\text{cof}(A)]^T = \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix} = \frac{1}{-30} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} 14{,}000 \\ 2{,}000 \end{bmatrix} \Rightarrow x = \begin{bmatrix} Q^* \\ P^* \end{bmatrix} = A^{-1} \cdot d$$

$$\begin{bmatrix} Q^* \\ P^* \end{bmatrix} = \frac{1}{-30} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 14{,}000 \\ 2{,}000 \end{bmatrix}$$

$$\begin{bmatrix} Q^* \\ P^* \end{bmatrix} = \begin{bmatrix} 10{,}000 \\ 400 \end{bmatrix}$$

∴ Equilibrium quantity of ice cream is 10,000 units. #

Equilibrium price of ice cream is 400 dollars per unit. #

Question 2:

✓ $C = C_0 + C_1 Y_d - C_2 r, \quad 0 < C_1 < 1$; $I_1 + C_1 - G_1 < 1.$

✓ $I = I_0 + I_1 Y - I_2 r, \quad 0 < I_1 < 1$

✓ $G = G_0 - G_1 Y, \quad 0 < G_1 < 1$

$\cancel{T = T_0}$ Exogenous variable

$\cancel{M^s = M_0}$ Exogenous variable

✓ $L^d = L_0 + L_1 Y - L_2 r$

a.) Since the equation of government in this question is $G = G_0 - G_1 Y$.
It could be interpreted that if Y (GDP) increases, the government spending will decrease.
Hence, it does not make sense in practice; naturally, the government spending will expand as the GDP increases.

b.) $C = C_0 + C_1 Y_d - C_2 r$
C_2 is the coefficient of r (interest rate) which is inversely related to consumption.
That is, when interest rate increases, the consumption will fall (due to substitution effect; people would save more).

c.) IS equation:

$$Y = C + I + G$$

$$Y = C_0 + C_1 Y_d - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$$

$Y_d = Y - T_0$; $$Y = C_0 + C_1 (Y - T_0) - C_2 r + I_0 + I_1 Y - I_2 r + G_0 - G_1 Y$$

$$Y = C_0 + \underline{C_1 Y} - C_1 T_0 - C_2 r + I_0 + \underline{I_1 Y} - I_2 r + G_0 - \underline{G_1 Y}$$

$$Y - C_1 Y - I_1 Y + G_1 Y = C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0$$

$$Y (1 - C_1 - I_1 + G_1) = C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0$$

$$Y = \frac{C_0 - C_1 T_0 - C_2 r + I_0 - I_2 r + G_0}{(1 - C_1 - I_1 + G_1)}$$

- slope of IS: $\frac{\Delta Y}{\Delta r} = \frac{-C_2 - I_2}{(1 - C_1 - I_1 + G_1)}$ ⇒ interest rate is negatively related to the level of GDP (Y) d.) When IS is flat, income (y) is highly sensitive to change in interest rate (r).

- slope of IS $= \dfrac{\Delta r}{\Delta y} = \dfrac{(1-C_1-I_1+G_1)}{-C_2-I_2}$

$$= \frac{\cancel{(-1)}(-1+C_1+I_1-G_1)}{\cancel{(-1)}(C_2+I_2)}$$

$$= \frac{-1+C_1+I_1-G_1}{C_2+I_2}$$

For IS to be flat, C_2+I_2 has to be large.

e.) Tax multiplier: $\dfrac{\Delta Y}{\Delta T_0} = \dfrac{-C_1}{(1-C_1-I_1+G_1)}$

When the slope of IS is flat (high MPC), the tax multiplier (negative MPC) will be larger, and vice versa.

f.) $\dfrac{M^s}{P} = L^d$

$$\frac{M_0}{P} = L_0 + L_1Y - L_2r$$

$$r = \frac{1}{\underbrace{L_2}_{\text{small: flat; large: steep}}}\left(L_0 + L_1Y - \frac{M_0^S}{P}\right)$$

g.) slope of LM: $\dfrac{\Delta r}{\Delta y} = \dfrac{L_1}{L_2}$

If L_1 is small or L_2 is high, LM curve is flat. This means the change in GDP (y) has small impact on the interest rate.

h.) Find Y and r

$$Y = C_0 + C_1Y - C_1T_0 - C_2r + I_0 + I_1Y - I_2r + G_0 - G_1Y$$

$$Y - C_1Y + C_2r - I_1Y + I_2r + G_1Y = C_0 - C_1T_0 + I_0 + G_0$$

$$(Y - C_1Y - I_1Y + G_1Y) + (C_2r + I_2r) = C_0 - C_1T_0 + I_0 + G_0$$

$$Y(1 - C_1 - I_1 + G_1) + r(C_2 + I_2) = C_0 - C_1T_0 + I_0 + G_0$$

$$r = \frac{1}{L_2}\left(L_0 + L_1Y - \frac{M_0^S}{P}\right)$$

$$r = \frac{L_0P + L_1PY - M_0^S}{L_2P}$$

$$-L_1PY + L_2Pr = L_0P - M_0^S$$

$$\underbrace{\begin{bmatrix} 1-C_1-z_1+G_1 & C_2+I_2 \\ -L_1P & L_2P \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y \\ r \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} C_0-C_1T_0+z_0+G_0 \\ L_0P-M^s_0 \end{bmatrix}}_{d}$$

i.) $$y^* = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} C_0-C_1T_0+z_0+G_0 & C_2+I_2 \\ L_0P-M^s_0 & L_2P \end{vmatrix}}{\begin{vmatrix} 1-C_1-z_1+G_1 & C_2+I_2 \\ -L_1P & L_2P \end{vmatrix}} = \frac{(C_0-C_1T_0+z_0+G_0)\cdot(L_2P))-((L_0P-M^s_0)\cdot(C_2+z_2))}{((1-C_1-z_1+G_1)(L_2P))-((-L_1P)\cdot(C_2+z_2))}$$

$$r^* = \frac{\det(A_1)}{\det(A)} = \frac{\begin{vmatrix} 1-C_1-z_1+G_1 & C_0-C_1T_0+z_0+G_0 \\ -L_1P & L_0P-M^s_0 \end{vmatrix}}{\begin{vmatrix} 1-C_1-z_1+G_1 & C_2+I_2 \\ -L_1P & L_2P \end{vmatrix}}$$

$$= \frac{((1-C_1-L_1+G_1)\cdot(L_0P-M^s_0))-((C_0-C_1T_0+z_0+G_0)\cdot(-L_1P))}{((1-C_1-z_1+G_1)(L_2P))-((-L_1P)\cdot(C_2+z_2))}$$

j.) $$y^* = \frac{(C_0-C_1T_0+z_0+G_0)\cdot(L_2P))-((L_0P-M^s_0)\cdot(C_2+z_2))}{((1-C_1-z_1+G_1)(L_2P))-((-L_1P)\cdot(C_2+z_2))}$$

$$r^* = \frac{((1-C_1-L_1+G_1)\cdot(L_0P-M^s_0))-((C_0-C_1T_0+z_0+G_0)\cdot(-L_1P))}{((1-C_1-z_1+G_1)(L_2P))-((-L_1P)\cdot(C_2+z_2))}$$

- $$\frac{\Delta y^*}{\Delta G_0} = \frac{L_2P}{((1-C_1-z_1+G_1)(L_2P))-((-L_1P)\cdot(C_2+z_2))}$$

$$= \frac{L_2P}{L_2P-L_2PC_1-L_2Pz_1+L_2PG_1+L_1PC_2+L_1Pz_2}$$

$$= \frac{L_2\cancel{P}}{\cancel{P}(L_2-L_2C_1-L_2z_1+L_2G_1+L_1C_2+L_1z_2)} = \frac{L_2}{L_2-L_2C_1-L_2z_1+L_2G_1+L_1C_2+L_1z_2}$$

- $$\frac{\Delta Y^*}{\Delta M_0} = \frac{C_2+I_2}{((1-C_1-I_1+G_1)\cdot(L_2P)) - ((-L_1P)\cdot(C_2+I_2))}$$

- $$\frac{\Delta r^*}{\Delta G_0} = \frac{L_1P}{((1-C_1-I_1+G_1)\cdot(L_2P)) - ((-L_1P)\cdot(C_2+I_2))}$$

$$= \frac{L_1P}{L_2P - L_2PC_1 - L_2PI_1 + L_2PG_1 + L_1PC_2 + L_1PI_2}$$

$$= \frac{L_1\cancel{P}}{\cancel{P}(L_2 - L_2C_1 - L_2I_1 + L_2G_1 + L_1C_2 + L_1I_2)} = \frac{L_1}{L_2 - L_2C_1 - L_2I_1 + G_1 + L_1C_2 + L_1I_2}$$

- $$\frac{\Delta r^*}{\Delta M_0} = \frac{-1+C_1+I_1-G_1}{((1-C_1-I_1+G_1)\cdot(L_2P)) - ((-L_1P)\cdot(C_2+I_2))}$$

It is given that $I_1+C_1-G_1<1$;

$$I_1+C_1-0<1$$

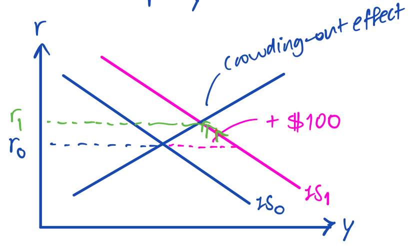
$$I_1+C_1<1$$

$$I_1+C_1-1<0$$

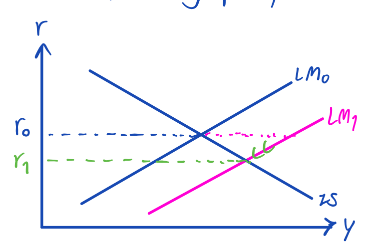
$$X(-1);\ 1-I_1-C_1>0$$

Suppose $G_1=0$, multiplier is bigger than the case that government is purely exogenous because it is given that $0<G<1$.

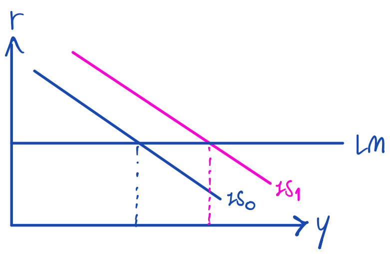
k.) Fiscal policy



Monetary policy



No, the government cannot achieve the first goal while keeping the interest rate constant (goal no. 2).



With only fiscal policy:

However, it is possible if the LM curve is flat (slope = 0) as the GDP rises while interest rate stays the same.

Change in GDP (y^*) = 100

- $$\frac{\Delta y^*}{\Delta G_0} = \frac{L_2}{(1-c_1-z_1+G_1)L_2 + (c_2+z_2)L_1}$$

$$\frac{100}{L_2} = \frac{\Delta G_0}{(1-c_1-z_1+G_1)L_2 + (c_2+z_2)L_1}$$

$$100 \cdot \frac{1}{L_2}\left[(1-c_1-z_1+G_1)L_2 + (c_2+z_2)L_1\right] = \Delta G_0$$

$$100\left[(1-c_1-z_1+G_1)\frac{\cancel{L_2}}{\cancel{L_2}}^{1} + (c_2+z_2)\left(\frac{L_1}{L_2}\right)\right] = \Delta G_0$$

must equal to 0 for LM to be flat

$$100(1-c_1-z_1+G_1) = \Delta G_0 \longrightarrow \frac{\Delta y^*}{\Delta G_0} = \frac{\cancel{100}}{\cancel{100}(1-c_1-z_1+G_1)}$$

$$= \frac{1}{(1-c_1-z_1+G_1)}$$

- $$\frac{\Delta y^*}{\Delta M_0} = \frac{c_2+z_2}{((1-c_1-z_1+G_1)\cdot(L_2P)) - ((-L_1P)\cdot(c_2+z_2))}$$

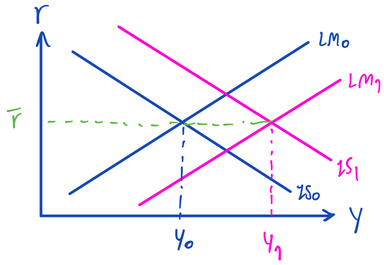
$$\frac{100}{c_2+z_2} = \frac{\Delta M_0}{(1-c_1-z_1+G_1)\cdot(L_2P) - (-L_1P)\cdot(c_2+z_2)}$$

$$100 \cdot \frac{1}{c_2+z_2}\left[(1-c_1-z_1+G_1)\cdot(L_2P) - (-L_1P)\cdot(c_2+z_2)\right] = \Delta M_0$$

$$100\left[(1-c_1-z_1+G_1)\cdot(L_2P)\cdot\frac{1}{c_2+z_2} + (L_1P)\cdot(\cancel{c_2+z_2})\cdot\frac{1}{\cancel{c_2+z_2}}\right] = \Delta M_0$$ sub into $\frac{\Delta y^*}{\Delta m}$;

$$\frac{\Delta y^*}{\Delta m} = \frac{\cancel{100}}{\cancel{100}\left[(1-c_1-z_1+G_1)\cdot(L_2P)\cdot\frac{1}{c_2+z_2} + (L_1P)\right]} = \frac{1}{\left[(1-c_1-z_1+G_1)\cdot(L_2P)\cdot\frac{1}{c_2+z_2} + (L_1P)\right]}$$

d.)



The government should use expansionary monetary policy to complement with the fiscal policy.

As the expansionary fiscal policy raises the interest rate, an expansionary monetary policy (eg: increase money supply) will keep interest rate constant.

[Money supply ↑ → excess supply of money (and excess demand for bond) → price of bond ↑ → interest rate ↓]