General Problem

① u is concave
agent is risk averse

② For Binary Case

$$f(w_a) = \max_{w_a} E\left[u(w_1)\right] = P_g\, u(w_1|g) + (1-P_g)\, u(w_1|B)$$

N possible outcomes of return on risky asset.

$$w_1|g = w_0(1+r_f) + w_a(r_g - r_f) \quad ; \quad r_g > r_f > r_B$$

risky good, risk free, risk bad

$$w_1|B = w_0(1+r_f) + w_a(r_B - r_f)$$

$$P_g(r_g) + (1-P_g) r_B > r_f$$

$\therefore$ FOC. $\dfrac{d\,E(u(w_1))}{d w_a} = 0$

$$\Rightarrow \underbrace{P_g \cdot \frac{du(w_1|g)}{d(w_1|g)} \cdot \frac{d(w_1|g)}{dw_a}}_{\text{upside gain}} + \underbrace{(1-P_g) \cdot \frac{du(w_1|B)}{d(w_1|B)} \cdot \frac{d(w_1|B)}{dw_a}}_{\text{downside loss}} = 0$$

$$E(MU(w_1)\cdot(r - r_f)) = 0$$

$u = 100W - 0.5W^2 \Rightarrow$ Increasing Absolute risk aversion

Increasing Relative risk aversion

$r_g = 0.1$

$r_f = 0.02$

$r_B = 0$

$W_0 = \overset{50}{\cancel{10}}$; $P_g = \frac{1}{2}$

$$\text{CARA} = \frac{1}{100-W}$$

$$\text{CRRA} = \frac{W}{100-W}$$

$\Rightarrow$

$$\tfrac{1}{2}(100 - W_1|g)(0.08) + \tfrac{1}{2}(100 - W_1|B)(-0.02) = 0$$

$$W_1|g = \overset{50}{\cancel{10}}(1+0.02) + W_a(0.08) = \overset{51}{\cancel{10.2}} + 0.08W_a$$

$$W_2|B = \overset{50}{\cancel{10}}(1+0.02) + W_a(-0.02) = \overset{51}{\cancel{10.2}} - 0.02W_a$$

$$\tfrac{1}{2}(\overset{49}{\cancel{89.8}} - 0.08W_a)(0.08) + \tfrac{1}{2}(\cancel{100} - \overset{51}{\cancel{10.2}} + 0.02W_a)(-0.02) = 0$$

$$\cancel{\tfrac{1}{2}}(\overset{49}{\cancel{89.8}} - 0.08W_a)(0.08) + \cancel{\tfrac{1}{2}}(\overset{49}{\cancel{89.8}} + 0.02W_a)(-0.02) = 0$$

Leverage Int mt

$W_a^* = 792.35$ (+) ; $W_0 - W_a^* = -782.35$ (−)

Borrowing with the risk free asset

$\underline{W_0 = 50} \Rightarrow W_a^* = 432.35$

$W_0 - W_a^* = 50 - 432.35 = -382.35$

Leverge

79.2

$\left.\begin{array}{l} W_0 \Rightarrow 10 \\ \Rightarrow W_0' = 50 \end{array}\right\}$ W_a^* ; $792.35 \Rightarrow$ **Preference**

$\downarrow$

$W_a^* = 432.35 \Rightarrow \frac{432.35}{50} = 8.67$ Increasig Absolute risk aversion

$\underline{W_0 = 10} \rightarrow CARA = \frac{1}{100-10} = \frac{1}{90}$

$CARA = \frac{1}{100-50} = \frac{1}{50}$

Investorn more risk averse as he has more wealth.

Leverage: increasing CRRA $\Rightarrow$ $CRRA = \frac{10}{90}$; $CRRA = \frac{50}{50} = 1$

$u = \ln(w)$; r_G ; r_B ; P_G ; $P_B = 1 - P_G$

$u' = \frac{1}{w}$ $\quad r_f$

$u'' = -\frac{1}{w^2}$ $\quad \rightarrow$ Initial wealth $= w_0$

$\downarrow$

- Determinants of the risky-asset investment

CARA $= -\frac{u''}{u'} = \frac{1}{w}$ $\quad w \uparrow \Rightarrow$ CARA $\downarrow$ $\quad$ 1,000 + ε

1,000 − ε

CRRA $=$ $w \cdot$ CARA $= 1 \Rightarrow$ Constant Value.

1000(1+ε)

1000(1−ε)

$w_0^* \rightarrow w_a^* \uparrow$ as $w_0 \uparrow$

$w_0^* \rightarrow \frac{w_a^*}{w_0}$ Constant $\rightarrow$ Elasticity of investment in risky asset to $w_0 = 1$

$$\max_{w_a} \quad E\,u(w_1) = P_g \cdot \ln(w_1|g) + (1-P_g) \cdot \ln(w_1|B)$$

$$w_1|g = w_0(1+r_f) + w_a(r_g - r_f)$$

$$w_1|B = w_0(1+r_f) + w_a(r_B - r_f)$$

$$\Rightarrow \quad P_g \cdot \frac{1}{w_1|g} \cdot (r_g - r_f) + (1-P_g) \cdot \frac{1}{w_1|B}(r_B - r_f) = 0$$

$P_g \cdot MU(w_1|g) \cdot$ (upside gain) $+ (1-P_g) \cdot MU(w_1|B)$ (Downside loss) $= 0$

$$\Rightarrow \quad P_g \cdot \frac{1}{w_0(1+r_f) + w_a(r_g - r_f)}(r_g - r_f) + (1-P_g) \cdot \frac{1}{w_0(1+r_f) + w_a(r_B - r_f)} \cdot (r_B - r_f) = 0$$

how to solve for $w_a = ?$

$$P_g\big(w_0(1+r_f) + w_a(r_B - r_f)\big)(r_g - r_f) + (1-P_g)\big(w_0(1+r_f) + w_a(r_g - r_f)\big) \cdot (r_B - r_f) = 0$$

$$P_g(w_o)(1+r_f)(r_g-r_f) + \underline{P_g \cdot w_a(r_B-r_f)(r_g-r_f)}$$
$$+ (1-P_g)(w_o(1+r_f)(r_B-r_f)) + \underline{(1-P_g)(w_a)(r_g-r_f)(r_B-r_f)} = 0$$

$r_g > r_f > r_B$

$$\underline{w_a(r_g-r_f)(r_B-r_f)}$$

move the rsh

$-w_a(r_g-r_f)\cdot(r_B-r_f)$

$$w_a(r_g-r_f)(r_f-r_B) = \cancel{P_g w_o(1+r_f) + (1-}$$

$$P_g w_o(1+r_f)(r_g-r_f)$$
$$+ (1-P_g) w_o(1+r_f)(r_B-r_f)$$

$$w_a = \frac{P_g(w_o)(1+r_f)(r_g-r_f) + (1-P_g)\cdot w_o(1+r_f)(r_B-r_f)}{(r_g-r_f)(r_f-r_B)}$$

$$W_a = \frac{\left[p_g (r_g - r_f) + (1-p_g)(r_B - r_f) \right] (1+r_f) \cdot W_0}{(r_g - r_f)(r_f - r_B)}$$

(i) $r_g > r_f > r_B$

(ii) $p_g r_g + (1-p_g)(r_B) > r_f$

$$p_g (r_g - r_f) + (1-p_g)(r_B - r_f)$$

$$\underbrace{p_g \cdot r_g + (1-p_g) \cdot r_B - r_f}_{\text{Excess Return} > 0}$$

(iii) $\frac{W_a}{W_0}$ = constant

$\Rightarrow$ independent of $\underline{\underline{W}}$

$\Rightarrow$ constant CRRA

$$W_a = \frac{\left[p_g \cdot r_g + (1-p_g) \cdot r_B - r_f \right] (1+r_f) \cdot W_0}{(r_g - r_f)(r_f - r_B)} \uparrow$$

$(r_g - r_f)(r_f - r_B) > 0$

always if $u''(\cdot) < 0$

(iv) $\underline{\underline{GR}} \uparrow \Rightarrow W_a \uparrow$

(v) Reduction fro $R_f \uparrow$ "risk of investing" $\Rightarrow W_a \downarrow$

(i). $\underline{W_a > 0}$

(ii) $\Rightarrow \frac{dW_a^*}{dW_0} = > 0 \rightarrow$ $\overset{C}{\cancel{R}}$ CARA is decreasing in $\underline{W_0}$

Any utility f^n

			W_a
→ Constant	CARA	$\Rightarrow W_0 \uparrow$	constant
Increasing			decrease
Decreasing			increase

			W_a/W_0
→ Constant	CRRA	$\Rightarrow W_0 \uparrow$	constant
In—			decrease
De—			increase