

EE435: Assignment 9

1. Perform unit root test of series y and x.

```
. dfuller y, trend lag(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 498

Test Statistic	Interpolated Dickey-Fuller			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(t)	1.000	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = **1.0000**

D.y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
y						
L1.	.0001178	.0001179	1.00	0.318	-.0001137	.0003494
LD.	.6997015	.0248993	28.10	0.000	.6507799	.7486231
_trend	2.897751	1.159296	2.50	0.013	.619992	5.175511
_cons	1811.233	147.0426	12.32	0.000	1522.327	2100.139

```
. dfuller d.y, trend lag(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 497

		----- Interpolated Dickey-Fuller -----		
	Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value

Z(t)	-10.554	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = 0.0000

D2.y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
D.y						
L1.	-.2787856	.0264156	-10.55	0.000	-.3306866	-.2268845
LD.	-.32127	.0373756	-8.60	0.000	-.3947051	-.2478349
_trend	3.631984	.3708318	9.79	0.000	2.903379	4.36059
_cons	1678.082	154.4788	10.86	0.000	1374.564	1981.6

```
. dfuller x, trend lag(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 498

Test Statistic	Interpolated Dickey-Fuller			
	1% Critical Value	5% Critical Value	10% Critical Value	
Z(t)	0.601	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = **0.9970**

D.x	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x						
L1.	.0001061	.0001764	0.60	0.548	-.0002405	.0004526
LD.	.46018	.0349881	13.15	0.000	.3914361	.5289239
_trend	4.166909	1.14105	3.65	0.000	1.924999	6.408818
_cons	2128.626	140.0551	15.20	0.000	1853.449	2403.803

To perform the unit root test, the hypothesis test is $H_0: \beta_1 = 1$. The result shows that p-value=1 which is higher than 0.05. Then, the null hypothesis of unit root is accepted at 95% level of significance. Hence, the series of y are non-stationary at I(0).

Since the series of y is non-stationary, we need to further perform the test whether the variables are integrated at level 1, I(1) or not. $H_0: \gamma = 0$. The result shows that p-value=0 which is lower than 0.05. Then, the null hypothesis of unit root is rejected at 95% level of significance. Hence, the series of y are integrated at level 1.

To perform the unit root test, the hypothesis test is $H_0: \beta_1 = 1$. The result shows that MacKinnon approximate p-value = 0.997 which is higher than 0.05. Then, the null hypothesis of unit root is accepted at 95% level of significance. Hence, the series of x are non-stationary at I(0).

```
. dfuller d.x, trend lag(1) regress
```

Augmented Dickey-Fuller test for unit root Number of obs = 497

----- Interpolated Dickey-Fuller -----				
Test	1% Critical	5% Critical	10% Critical	
Statistic	Value	Value	Value	
Z(t)	-10.657	-3.980	-3.420	-3.130

MacKinnon approximate p-value for Z(t) = 0.0000

D2.x	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
D.x					
L1.	-.4007114	.0376021	-10.66	0.000	-.4745915 -0.3268312
LD.	-.414172	.0358603	-11.55	0.000	-.4846298 -0.3437141
_trend	3.508317	.3506567	10.00	0.000	2.819351 4.197283
_cons	1596.429	147.0971	10.85	0.000	1307.415 1885.444

Since the series of x is non-stationary, we need to further perform the test whether the variables are integrated at level 1, $I(1)$ or not. $H_0: \gamma = 0$. The result shows that $p\text{-value}=0$ which is lower than 0.05 . Then, the null hypothesis of unit root is rejected at 95% level of significance. Hence, the series of x are integrated at level 1.

2. Perform cointegration test of series y and x using set up of (i) linear trend; (ii) restricted trend; (iii) unrestricted constant; (iv) restricted constant; and (v) no trend, with one lag term.

Linear trend

```
. vecrank y x, trend(t) lags(1) max
```

Johansen tests for cointegration						
Trend: trend			Number of obs =		499	
Sample: 2 - 500			Lags =		1	
maximum rank	parms	LL	eigenvalue	trace statistic	5% critical value	
0	4	-7387.4577	.	790.3357	18.17	
1	7	-6992.3441	0.79477	0.1085*	3.74	
2	8	-6992.2899	0.00022			
maximum rank	parms	LL	eigenvalue	max statistic	5% critical value	
0	4	-7387.4577	.	790.2272	16.87	
1	7	-6992.3441	0.79477	0.1085	3.74	
2	8	-6992.2899	0.00022			

1. $H_0: r = 0$ $H_a: r \geq 1$

2. $H_0: r = 1$ $H_a: r \geq 2$

Since the trace statistic of rank 0 (790.3357) is higher than the critical value (18.17), the null hypothesis is rejected. Then, in rank1, the trace statistic (0.1085) is lower than the critical value (3.74). The null hypothesis is accepted at 95% level of significance. Rank is 1

Restricted trend

```
. vecrank y x, trend(rt) lag(1/1) max
```

Johansen tests for cointegration						
Trend: rtrend			Number of obs =		499	
Sample: 2 - 500			Lags =		1	
maximum rank	parms	LL	eigenvalue	trace statistic	5% critical value	
0	2	-8050.4781	.	2116.3764	25.32	
1	6	-7075.2453	0.97993	165.9107	12.25	
2	8	-6992.2899	0.28286			
maximum rank	parms	LL	eigenvalue	max statistic	5% critical value	
0	2	-8050.4781	.	1950.4657	18.96	
1	6	-7075.2453	0.97993	165.9107	12.52	
2	8	-6992.2899	0.28286			

Since the trace statistic of rank 0 (2116.3764) is higher than the critical value (25.32), the null hypothesis is rejected. Then, in rank1, the trace statistic (165.9107) is higher than the critical value (12.25). The null hypothesis is rejected at 95% level of significance. Rank is 2

Unrestricted constant

```
. vecrank y x, trend(c) lags(1/1) max
```

Johansen tests for cointegration					
Trend: constant			Number of obs =		499
Sample: 2 - 500			Lags =		1

maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical
0	2	-8050.4781	.	2086.8946	15.41
1	5	-7083.8611	0.97923	153.6607	3.76
2	6	-7007.0308	0.26504		

maximum				max	5%
rank	parms	LL	eigenvalue	statistic	critical
0	2	-8050.4781	.	1933.2339	14.07
1	5	-7083.8611	0.97923	153.6607	3.76
2	6	-7007.0308	0.26504		

Since the trace statistic of rank 0 (2086.8946) is higher than the critical value (15.41), the null hypothesis is rejected. Then, in rank1, the trace statistic (153.6607) is higher than the critical value (3.765). The null hypothesis is rejected at 95% level of significance. Rank is 2

Restricted constant

```
. vecrank y x, trend(rc) lags(1/1) max
```

Johansen tests for cointegration					
Trend: rconstant			Number of obs =		499
Sample: 2 - 500			Lags =		1

maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical
0	0	-8811.3759	.	3607.1930	19.96
1	4	-7092.1887	0.99898	168.8185	9.42
2	6	-7007.7794	0.28703		

maximum				max	5%
rank	parms	LL	eigenvalue	statistic	critical
0	0	-8811.3759	.	3438.3745	15.67
1	4	-7092.1887	0.99898	168.8185	9.24
2	6	-7007.7794	0.28703		

Since the trace statistic of rank 0 (3607.1930) is higher than the critical value (19.96), the null hypothesis is rejected. Then, in rank1, the trace statistic (168.8185) is higher than the critical value (9.42). The null hypothesis is rejected at 95% level of significance. Rank is 2

No trend

```
. vecrank y x, trend(n) lags(1/1) max
```

Johansen tests for cointegration					
Trend: none			Number of obs =		499
Sample: 2 - 500			Lags =		1

maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical
0	0	-8811.3759	.	3204.1194	12.53
1	3	-7211.7592	0.99836	4.8860	3.84
2	4	-7209.3162	0.00974		

maximum				max	5%
rank	parms	LL	eigenvalue	statistic	critical
0	0	-8811.3759	.	3199.2334	11.44
1	3	-7211.7592	0.99836	4.8860	3.84
2	4	-7209.3162	0.00974		

Since the trace statistic of rank 0 (3204.1194) is higher than the critical value (12.53), the null hypothesis is rejected. Then, in rank1, the trace statistic (4.886) is higher than the critical value (3.84). The null hypothesis is rejected at 95% level of significance. Rank is 2

3. Perform cointegration test of series y and x using set up of linear trend with (i) one lag term; (ii) two lag terms; and (iii) three lag terms.

Linear trend with 1 lag term

```
. vecrank y x, trend(t) lags(1) max
```

Johansen tests for cointegration					
Trend: trend			Number of obs =		499
Sample: 2 - 500			Lags =		1
maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical
0	4	-7387.4577	.	790.3357	18.17
1	7	-6992.3441	0.79477	0.1085*	3.74
2	8	-6992.2899	0.00022		

maximum				max	5%
rank	parms	LL	eigenvalue	statistic	critical
0	4	-7387.4577	.	790.2272	16.87
1	7	-6992.3441	0.79477	0.1085	3.74
2	8	-6992.2899	0.00022		

Linear trend with 2 lag terms

```
. vecrank y x, trend(t) lags(2) max
```

Johansen tests for cointegration					
Trend: trend			Number of obs =		498
Sample: 3 - 500			Lags =		2
maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical
0	8	-6795.5337	.	187.5771	18.17
1	11	-6703.3826	0.30932	3.2749*	3.74
2	12	-6701.7451	0.00655		

maximum				max	5%
rank	parms	LL	eigenvalue	statistic	critical
0	8	-6795.5337	.	184.3022	16.87
1	11	-6703.3826	0.30932	3.2749	3.74
2	12	-6701.7451	0.00655		

Linear trend with 3 lag terms

```
. vecrank y x, trend(t) lags(3) max
```

Johansen tests for cointegration					
Trend: trend			Number of obs =		497
Sample: 4 - 500			Lags =		3
maximum				trace	5%
rank	parms	LL	eigenvalue	statistic	critical
0	12	-6756.658	.	140.0434	18.17
1	15	-6688.113	0.24106	2.9534*	3.74
2	16	-6686.6363	0.00592		

maximum				max	5%
rank	parms	LL	eigenvalue	statistic	critical
0	12	-6756.658	.	137.0900	16.87
1	15	-6688.113	0.24106	2.9534	3.74
2	16	-6686.6363	0.00592		

4. From (2) & (3), determine the most appropriated set up and make the conclusion of the test whether y and x are cointegrated series? If yes, how many cointegrating Equations.

1. $H_0: r = 0$ $H_a: r \geq 1$

2. $H_0: r = 1$ $H_a: r \geq 2$

With one lag term, since the trace statistic of rank 0 (790.3357) is higher than the critical value (18.17), the null hypothesis is rejected. Then, in rank1, the trace statistic (0.1085) is lower than the critical value (3.74). The null hypothesis is accepted at 95% level of significance. The rank is 1 with 1 lag term. Thus, all variables are stationary. There is no cointegrating equation because of the integrating VAR at 0.

With two lag terms, since the trace statistic of rank 0 (187.5771) is higher than the critical value (18.17), the null hypothesis is rejected. Then, in rank1, the trace statistic (3.2749) is lower than the critical value (3.74). The null hypothesis is accepted at 95% level of significance. The rank is 1 with 2 lags term. Thus, the cointegrating equation exists and at most 1.

With three lag terms, since the trace statistic of rank 0 (140.0434) is higher than the critical value (18.17), the null hypothesis is rejected. Then, in rank1, the trace statistic (2.9534) is lower than the critical value (3.74). The null hypothesis is accepted at 95% level of significance. The rank is 1 with 3 lags term. Thus, the cointegrating equation exists and at most 1.

5. Estimated VECM models of y and x using (i) one lag term; (ii) two lag terms; and (iii) three lag terms. Determine the optimal lags. Specify cointegrating equation and speed of adjustment parameters.

From the data, the overall test for cointegrated equations is significant at 95% level since all $P > \chi^2$ values (0.00) is less than 0.05, indicating the cointegrated relationship.

To determine the optimal lag, we need to consider the SBIC value. The optimal lag is 3 since it has the lowest SBIC of 27.23459.

Cointegrating equation: $y_t = \beta_0 + \beta_1 x_t$. From, $y_t - 1.50049x_t - 92.13439 = 0$

Then, $y_t = 92.13439 + 1.50049x_t$ is the cointegrating equation for lag-3 model

The speed of adjustment parameters of y is -0.3193909 and x is 0.4434064 in the 3-lag model.

One lag term

. vec y x, lags(1)

Vector error-correction model

Sample: 2 - 500
Number of obs = 499
AIC = 28.41227
Log likelihood = -7083.861
HQIC = 28.42883
Det(Sigma_ml) = 7.34e+09
SBIC = 28.45448

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	2	331.479	0.9988	399343.6	0.0000
D_x	2	430.726	0.9953	105050.2	0.0000

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
D_y	_ce1					
	L1.	-1.303404	.0095397	-136.63	0.000	-1.322102 -1.284707
	_cons	-107.843	69.40373	-1.55	0.120	-243.8718 28.18583
D_x	_ce1					
	L1.	-.8364345	.0123959	-67.48	0.000	-.8607301 -.812139
	_cons	168.0502	90.1839	1.86	0.062	-8.706965 344.8074

Cointegrating equations

Equation	Parms	chi2	P>chi2
_ce1	1	1.71e+10	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

beta		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
_ce1						
	y	1	.	.	.	.
	x	-1.500508	.0000115	-1.3e+05	0.000	-1.500531 -1.500486
	_cons	-1678.204	.	.	.	.

Two lag terms

. vec y x, lags(2)

Vector error-correction model

Sample: 3 - 500
Number of obs = 498
AIC = 27.18032
Log likelihood = -6758.899
HQIC = 27.21018
Det(Sigma_ml) = 2.11e+09
SBIC = 27.25641

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	4	213.415	0.9995	964110.3	0.0000
D_x	4	263.876	0.9982	280728.2	0.0000

		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
D_y	_ce1					
	L1.	-.427199	.0483639	-8.83	0.000	-.5219905 -.3324075
	L2.	-.0022605	.0224005	17.96	0.000	-.3583563 .4461646
	y					
	LD.	.4814843	.0598769	8.04	0.000	.3641276 .5988409
	_cons	171.4547	44.03061	3.89	0.000	85.15624 257.7531
D_x	_ce1					
	L1.	.2903827	.0597995	4.86	0.000	.1731779 .4075875
	L2.	.5734396	.0276971	20.70	0.000	.5191544 .6277249
	y					
	LD.	.3676105	.0740347	4.97	0.000	.2225051 .5127159
	_cons	252.237	54.44157	4.63	0.000	145.5335 358.9405

Cointegrating equations

Equation	Parms	chi2	P>chi2
_ce1	1	8.09e+08	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

beta		Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
_ce1						
	y	1	.	.	.	.
	x	-1.500176	.0000527	-2.8e+04	0.000	-1.50028 -1.500073
	_cons	-577.9622	.	.	.	.

Three lag terms

. vec y x, lags(3)

Vector error-correction model

Sample: 4 - 500
Log likelihood = -6727.439
Det(Sigma_ml) = 1.96e+09
Number of obs = 497
AIC = 27.1245
HQIC = 27.16771
SBIC = 27.23459

Equation	Parms	RMSE	R-sq	chi2	P>chi2
D_y	6	211.673	0.9995	980003.4	0.0000
D_x	6	253.098	0.9984	305162.2	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
D_y						
_cel						
L1.	-.3193909	.058155	-5.49	0.000	-.4333726	-.2054092
y						
LD.	.3553209	.0559202	6.35	0.000	.2457194	.4649225
L2D.	.0174858	.0312998	0.56	0.576	-.0438607	.0788323
x						
LD.	.6042668	.0763396	7.92	0.000	.4546439	.7538896
L2D.	.0521960	.0634683	0.82	0.411	-.0721988	.1765925
_cons	181.5838	45.38519	4.00	0.000	92.63049	270.5372
D_x						
_cel						
L1.	.4434064	.0695359	6.38	0.000	.3071185	.5796943
y						
LD.	.4713427	.0668638	7.05	0.000	.3402922	.6023932
L2D.	.012763	.0374252	0.34	0.733	-.060589	.0861151
x						
LD.	.5220109	.0912792	5.72	0.000	.3431069	.700915
L2D.	.0911602	.0758891	1.20	0.230	-.0575797	.2399001
_cons	130.797	54.26707	2.41	0.016	24.43548	237.1585

Cointegrating equations

Equation	Parms	chi2	P>chi2
_cel	1	6.64e+08	0.0000

Identification: beta is exactly identified

Johansen normalization restriction imposed

beta	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
_cel						
y	1	.	.	.	.	.
x	-1.500049	.0000582	-2.6e+04	0.000	-1.500163	-1.499935
_cons	-92.13439	.	.	.	.	.