

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.), 2020

6004641061

Use 4 decimal places for numerical answers

1. In Table 1.a, X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$y - \bar{y}$	$x_i - \bar{x}$
-0.9125	-14.625
0.1875	-5.625
-0.9125	0.375
0.2875	3.375
0.3875	9.375
-0.2125	-2.625
-0.5125	-2.625
0.4875	12.375

$$\bar{x} = \frac{63+72+78+81+87+75+75+90}{8}$$

$$= 77.625$$

$$\bar{y} = \frac{2.8+3.4+3+3.5+3.6+3+2.7+3.7}{8}$$

$$= 3.2125$$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1 . (Note: $NIID$ = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{(63-77.625)(2.8-3.2125) + (72-77.625)(3.4-3.2125) + \dots + (90-77.625)(3.7-3.2125)}{(-14.625)^2 + (-5.625)^2 + \dots + (12.375)^2}$$

$$= \frac{17.9375}{511.875} = 0.0341$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 3.2125 - (0.0341)(77.625)$$

$$\hat{\beta}_0 = 0.5659$$

1.2 For each observation i , find $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=1}^n \hat{u}_i = 0$.

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i = 0.5659 + 0.0341 x_i$$

$$\hat{u}_i = y_i - \hat{y}_i = y_i - 0.5659 - 0.0341 x_i$$

$$\sum_{i=1}^n \hat{u}_i = (2.8 - 0.5659 - 0.0341(63)) + (3.4 - 0.5659 - 0.0341(72)) + \dots + (3.7 - 0.5659 - 0.0341(90))$$

$$= -0.0002$$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

$$SST = \sum (x_i - \bar{x})^2 = 951.875$$

$$\sum x_i^2 = 48717$$

$$SSR = \sum (\hat{y}_i - \bar{y})^2 = 1.02875$$

$$\sigma^2 = var(u_i | x_i) = E[(u_i - E(u_i))^2] = E(u_i^2)$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{\sigma^2}{951.875} = 0.003$$

$$var(\hat{u}_i) = \sigma^2 = \frac{SSR}{n-2} = \frac{1.02875}{8-2} = 0.1715$$

$$var(\hat{\beta}_0) = \sigma^2 \frac{1}{n-2} \sum x_i^2$$

$$= \frac{10.1115 \cdot 48717}{8 \cdot 951.875} = 2.0398$$

2. Data is listed in the table

X_i	Y_i	$x_i - \bar{x}$	$y_i - \bar{y}$	$\hat{u}_i$
10	0	-10	-9.1	-0.1455
12	2	-8	-7.1	0.0636
14	5	-6	-4.1	1.2727
16	6	-4	-3.1	0.4818
18	7	-2	-2.1	-0.3091
22	10	2	0.9	-0.4909
24	10	4	0.9	-2.6818
26	15	6	5.9	0.5273
28	16	8	6.9	-0.2636
30	20	10	10.9	1.9455

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\bar{x} = (10 + 12 + 14 + 16 + 18 + 22 + 24 + 26 + 28 + 30) / 10 = 20$$

$$\bar{y} = (0 + 2 + 5 + 6 + 7 + 10 + 10 + 15 + 16 + 20) / 10 = 9.1$$

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{(-10)(-9.1) + (-8)(-7.1) + \dots + (10)(10.9)}{(-10)^2 + (-8)^2 + \dots + 10^2} = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 9.1 - 0.8955(20) = -8.01$$

2.2 Find the value of $\hat{y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$.

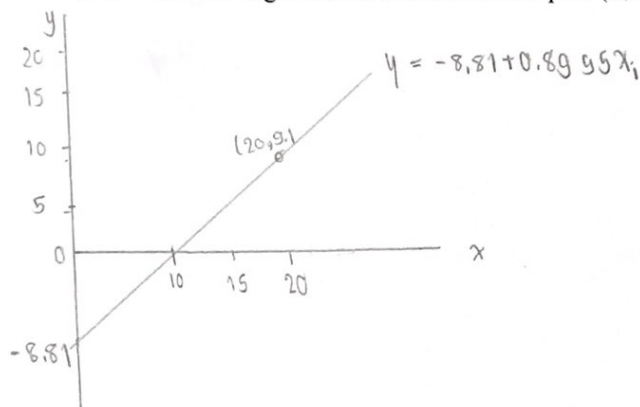
$$\hat{y}_i = \beta_0 + \beta_1 x_i = -8.81 + 0.8955 x_i$$

$$u_i = y_i - \hat{y}_i = y_i + 8.81 - 0.8955 x_i$$

$$\sum_{i=0}^N \hat{u}_i = (10 + 8.81 - 0.8955(10)) + (2 + 8.81 - 0.8955(12)) + \dots + (20 + 8.81 - 0.8955(30))$$

$$= 0$$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



Yes, it pass $(\bar{x}, \bar{y})$

2.4 If $X_i = 16$, what is the predicted Y ?

$$E(y) = -8.81 + 0.8955(16)$$

$$= 5.518$$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$ $SSR = \sum (y - \hat{y})^2 = 366.9$
 $SST = \sum (x - \bar{x})^2 = 440$

$$var(\hat{u}_i) = \frac{SSR}{n-2} = \frac{366.9}{10-2} = 45.8625$$

$$var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^n x_i^2}{SST}$$

$$= \frac{(45.8625)(440)}{10(440)}$$

$$= 45.8625$$

$$var(\hat{\beta}_1) = \frac{\sigma^2}{SST}$$

$$= \frac{45.8625}{440}$$

$$= 0.104233$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$y_i = \beta_1 x_i + u_i$$

$$\text{argmin } \sum_i (y_i - \tilde{\beta}_1 x_i)^2$$

$$\text{F.O.C. wr.t. } \tilde{\beta}_1$$

$$\frac{\partial \sum_i (y_i - \tilde{\beta}_1 x_i)^2}{\partial \tilde{\beta}_1} = 0$$

$$\sum_i 2(y_i - \tilde{\beta}_1 x_i)(-x_i) = 0$$

$$\sum_i x_i (y_i - \tilde{\beta}_1 x_i) = 0 \quad (-2)$$

$$\sum_i x_i y_i - \sum_i \tilde{\beta}_1 x_i^2 = 0$$

$$\sum_i x_i y_i = \sum_i \tilde{\beta}_1 x_i^2$$

$$\tilde{\beta}_1 = \frac{\sum_i x_i y_i}{\sum_i x_i^2}$$

$$\text{Substitute } y_i = \beta_1 x_i + u_i$$

$$\hat{\beta}_1 = \frac{\sum_i x_i (\beta_1 x_i + u_i)}{\sum_i x_i^2}$$

$$\hat{\beta}_1 = \frac{\sum_i \beta_1 x_i^2}{\sum_i x_i^2} + \frac{\sum_i u_i x_i}{\sum_i x_i^2}$$

$$E(\hat{\beta}_1) = E(\beta_1) + E\left(\frac{\sum_i u_i x_i}{\sum_i x_i^2}\right) \quad \text{by SLR 4} \quad E(u_i | x_i) = 0$$

$$E(\hat{\beta}_1) = \beta_1$$