**Assignment #1**

**Instructions:**

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

**1.** Given this information

| | | | | |
|---|---|---|---|---|
| $n = 30$ | $\sum_{i=1}^{n} X_i = 366$ | $\sum_{i=1}^{n} Y_i = 631$ | $\overline{X} = 12.20$ | $\overline{Y} = 21.03$ |
| $\sum_{i=1}^{n} (X_i)^2 = 5,564$ | $\sum_{i=1}^{n} X_i Y_i = 7,524$ | $\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$ | $\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$ | |
| $\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$ | | $\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$ | | |

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of $\beta_1$ and $\beta_2$ with OLS method and explain the meaning of the model.

b) Find $r^2$ and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\operatorname{var}(u_i)$, $\operatorname{var}\left(\hat{\beta}_1\right)$ and $\operatorname{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

**2.** Given that $Y$ is market price of a car (USD) while $X$ is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = \overset{y_i}{7{,}836} - \overset{\beta_2}{502.4}X_i$$
$$(52) \quad (411.8)$$

Given that $u_i$ is normally, identically and independently distributed with zero mean and $\sigma^2$ variance, total number of observations is $\underset{n}{11}$,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the $X$ with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

1.a. OLS method

$\beta_1 = \bar{Y} - \beta_2 \bar{X} = 21.03 - (-0.1585)(12.20) = 22.9637$

$\beta_2 = \dfrac{\Sigma x_i y_i}{\Sigma x_i^2} = \dfrac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2} = \dfrac{-174.20}{1098.8} \approx -0.1585$

As a result of $\beta_1 = 22.9637$, it means that $Y_i = 22.9637$ when $X_i = 0$.
$\beta_2 = -0.1585$, it means that $Y_i$ will be increased by $-0.1585$ after $X_i$ increasing by 1.

1.b. $r^2 = \dfrac{ESS}{TSS} = 1 - \dfrac{RSS}{TSS} = 1 - \dfrac{\Sigma \hat{u}_i^2}{\Sigma (Y_i - \bar{Y})^2} = 1 - \dfrac{873.14}{882.97} = 0.0111$

Hence, 0.0111 of the variation in Y is explained by the variation in X.
While, the remaining 0.9889 is unexplained as residual.

1.c. $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_1 = 22.9637 + (-0.1585)(5) = 22.1712$

Therefore, $\hat{Y}_i$ will be 22.1712 when $X_i = 5$

1.d. $var(u_i) = \hat{\sigma}^2 = \dfrac{\Sigma \hat{u}_i^2}{n-k} = \dfrac{873.14}{30-2} = 31.1835$

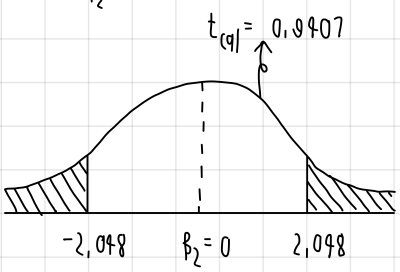
$\Sigma x_i^2 = \Sigma (X_i - \bar{X})^2 = 1098.8$

$var(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1} = \dfrac{\Sigma X_i^2}{n \Sigma x_i^2} \sigma^2 = \dfrac{(5564)(31.1835)}{(30)(1098.8)} = 5.2634$

$var(\hat{\beta}_2) = \sigma^2_{\hat{\beta}_2} = \dfrac{\sigma^2}{\Sigma x_i^2} = \dfrac{31.1835}{1098.8} = 0.2840$

1.e. $H_0 : \beta_2 = 0$ null hypothesis
$\beta_2 \neq 0$ alternative hypothesis
$\alpha = 0.05$ d.f. = 28

$t = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{-0.1585 - 0}{0.1085} = 0.9407$

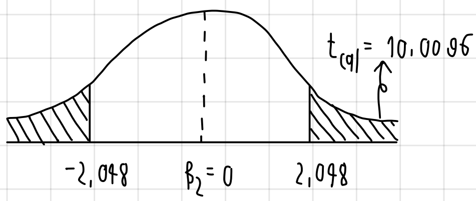



the lower bound $t_{\alpha/2} = -2.048$
the upper bound: $t_{\alpha/2} = 2.048$

∴ $t_{cal}$ is in the areas of acceptance region, so $H_0$ is not rejected.

$H_0 : \beta_1 = 0$ null hypothesis
$\beta_1 \neq 0$ alternative hypothesis
$\alpha = 0.05$ d.f. = 28

$t = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{22.9641 - 0}{2.2942} = 10.0096$



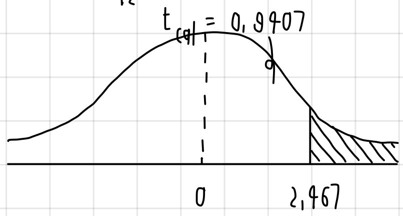

the lower bound $t_{\alpha/2} = -2.048$
the upper bound: $t_{\alpha/2} = 2.048$

∴ $t_{cal}$ is in the area of rejection region, so $H_0$ is rejected.

1.f.

$H_0: \beta_2 \leqslant 0$ null hypothesis
$H_1: \beta_2 > 0$ alternative hypothesis
$\alpha = 0.01$ $d.f. = 28$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\beta_2}} = \frac{0.1585 - 0}{0.1085} = 0.9407$$

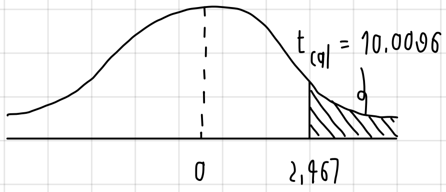



the upper bound: $t_{0.01} = 2.467$

$\therefore t_{cal}$ is in the area of acceptance region, so $H_0$ is not rejected.

$H_0: \beta_1 \leqslant 0$ null hypothesis
$H_1: \beta_1 > 0$ alternative hypothesis
$\alpha = 0.01$ $d.f. = 28$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\beta_2}} = \frac{22.9641 - 0}{2.2942} = 10.0096$$




the upper bound: $t_{0.01} = 2.467$

$\therefore t_{cal}$ is in the area of rejection region, so $H_0$ is rejected.

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2.a. Yes, the regression function is negative slope when X (car aged) increases, Y (market price) will be decreased.

2.b. $E(Y \mid X_0 = 5) = 7{,}830 - 502.4(5)$

$\hat{Y}_0 = 7{,}830 - 2{,}512 = 5{,}324$

$var(\hat{Y}_0) = \sigma^2 [1/n + (X_0 - \bar{X})^2 / \Sigma(X_i - \bar{X})^2]$

$\sigma^2(\hat{Y}_0) = (212{,}877)[1/11 + (5 - 7.45)^2 / 78.73]$

$= 35{,}582.5355$

$\sigma(\hat{Y}_0) = 188.0333$

given $\alpha = 0.05$

$n - k = 11 - 2 = 9$

$Pr[5.324 - 2.202(188.0333) \leqslant Y_0 \leqslant 5.324 + 2.202(188.0333)]$

$= Pr[4{,}897.3115 \leqslant Y_0 \leqslant 5{,}750.6885]$ ; $Y_0$ = 5 years old car market price

$= 0.95$ or $95\%$

2.c. SRF ; $\hat{Y}_i = 7830 - 50.24(10X)$
se (52) (41.18)

2.d. $X = 10$ $Y = 2{,}812$

$$\frac{dy}{dx} \cdot \frac{X}{Y} = 502.4 \cdot \frac{10}{2{,}812} = 1.7866$$