

MA 332 Linear Algebra
Exercise V

1. Let $v = (-1, 1, 2)$ and form the rank one matrix

$$vv^T = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} -1 & 1 & 2 \end{bmatrix}.$$

Describe the nullspace of vv^T geometrically.

2. Find a basis for $W^\perp$ where

$$W = \text{span} \left(\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \right).$$

3. Find the orthogonal projection of

$$v = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

onto the subspace of $\mathbf{R}^4$ spanned by

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}.$$

4. Let S be the vector space spanned by vectors $(1, 0, -2, 0)$, $(0, 2, 4, -1)$ and $(2, 2, 0, -1)$. Find $S^\perp, (S^\perp)^\perp$. What is the relationship between $(S^\perp)^\perp$ and S ?

5. Find the projection of b onto the column space of A by solving $A^T A \hat{x} = A^T b$ and $p = A \hat{x}$.

$$\text{a) } A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 1 & 3 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}. \quad \text{b) } A = \begin{bmatrix} 0 & 2 \\ -1 & 3 \\ 2 & -1 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

6. Find the least squares parabola for the data points $\{(1, 2), (2, 5), (3, 7), (4, 1)\}$.