pavita kriathkungwalkai
6104640674

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\bar{x} = \frac{63+72+78+81+87+75+75+90}{8}$$

$\bar{x} = 77.625$

$$\bar{y} = \frac{2.8+3.4+3+3.5+3.6+3.0+2.7+3.7}{8}$$

$\bar{y} = 3.2125$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\Sigma_i (x_i - \bar{x})(y_i - \bar{y})}{\Sigma_i (x_i - \bar{x})^2}$$

$$= \frac{(63-77.625)(2.8-3.2125)+(72-77.625)(3.4-3.2125)+(78-77.625)(3-3.2125) +(81-77.625)(3.5-3.2125)+(87-77.625)(3.6-3.2125)+(75-77.625)(3-3.2125) +(75-77.625)(2.7-3.2125)+(90-77.625)(3.7-3.2125)}{(63-77.625)^2+(72-77.625)^2+(78-77.625)^2+(81-77.625)^2+(87-77.625)^2 +(75-77.625)^2+(75-77.625)^2+(90-77.625)^2}$$

$$= \frac{17.4374}{511.875}$$

$= 0.0341$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$= 3.2125 - (0.0341)(77.625)$

$= 0.5655$

1.2 For each observation i, find $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

y	x	$\hat{y}$	$\hat{u}$
2.8	63	2.7143	0.0857
3.4	72	3.0209	0.3791
3	78	3.2253	-0.2253
3.5	81	3.3275	0.1725
3.6	87	3.5319	0.0651
3	75	3.1231	-0.01231
2.7	75	3.1231	-0.4231
3.7	90	3.6341	0.0639

calculated by $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$
$\hat{u}_i = y_i - \hat{y}_i$

$\Sigma_{i=0}^{N} \hat{u}_i = 0$

$\hat{u} = y_i - \bar{y}$

$= (2.8-3.2125)+(3.4-3.2125)+(3-3.2125)+(3.5-3.2125)$
$+(3.6-3.2125)+(3-3.2125)+(2.7-3.2125)+(3.7-3.2125)$
$= 0$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$\bar{X} = \frac{10+12+14+16+18+22+24+26+28+30}{10}$

$= 20$

$\bar{Y} = \underline{0+2+5+6+7+10+10+15+16+20}$

$= 9.1$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$\hat{\beta}_1 = \frac{394}{440} = 0.8955$

$\hat{\beta}_0 = 9.1 - (0.8955)(20)$

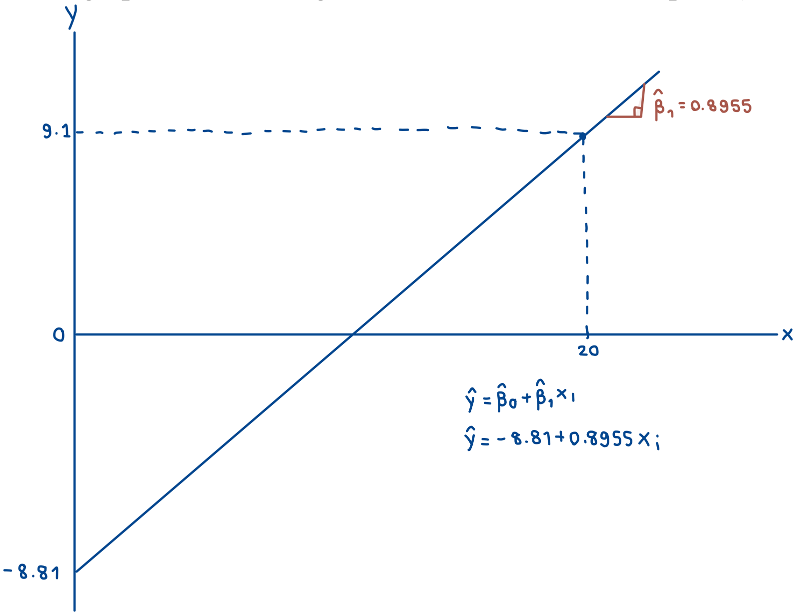
$= -8.81$

2.2 Find the value of $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

x_i	y_i	$\hat{y}_i$	$\hat{u}_i$
10	0	0.145	-0.145
12	2	1.936	0.064
14	5	3.727	1.273
16	6	5.518	0.482
18	7	7.309	-0.309
22	10	10.891	-0.891
24	10	12.682	-2.682
26	15	14.473	0.527
28	16	16.264	-0.264
30	20	18.055	1.945

$\sum_{i=0}^{N} \hat{u}_i = \underline{\underline{0}}$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_i$

$\hat{y} = -8.81 + 0.8955 x_i$

estimated $y = \hat{y}$

2.4 If X_i =16, what is the predicted Y?

$x_i = 16$, $\hat{y} = 5.518$

2.5 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$0 = \Sigma_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$$

$$= \Sigma_1 y_i - n\hat{\beta}_0 - \hat{\beta}_1 \Sigma x_i$$

$$\hat{\beta}_0 = \frac{\Sigma_i y_i}{n} - \frac{\hat{\beta}_1 \Sigma x_1}{n}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

But in this case $\hat{\beta}_0 = 0$

$$0 = \Sigma_i x_i (y_i - (0) - \hat{\beta}_1 x_i)$$

$$= \Sigma_i x_i y_1 - \Sigma_i \hat{\beta}_i x_i^2$$

$$\hat{\beta}_1 = \Sigma_i x_i (y_i - \hat{\beta}_i x_i)$$