

## Multinomial Logit & Ordered Probit Models

### Multinomial Logit Model

Dependent variable has more than two choices.

$$p_{ij} = \frac{e^{x_i' \beta_j}}{\sum_{h=1}^3 e^{x_i' \beta_h}} = \frac{e^{x_i' \beta_j}}{1 + \sum_{h=2}^3 e^{x_i' \beta_h}}$$

### Ordered Probit Model

In case that choices are ordered, ordered probit model can be applied.

$$y_i^* = x_i' \beta + \varepsilon_i, \quad E(\varepsilon_i) = 0$$

$$y_i = 1 \quad \text{if } -\infty < y_i^* \leq c_1,$$

$$y_i = 2 \quad \text{if } c_1 < y_i^* \leq c_2,$$

$$y_i = 3 \quad \text{if } c_2 < y_i^* \leq \infty$$

where:  $c_i$  = Threshold value and  $i=1,2$

### Data

Dependent Variables--Financing Choices (y)

1 = Internal Finance

2 = Long- term debt and bond

3 = Share issues

Independent Variables ( $x_k$ ):

$x_1$  = Liquidity

$x_2$  = Profitability

$x_3$  = Log of firm size

$x_4$  = Interest Payments

$x_6$  = Depreciation

$x_7$  = Deviation from target capital structure

**Multinomial Logit Model**

```
. mlogit y x1 x2 x3 x4 x5 x6 x7
```

```
Iteration 0:   log likelihood =  -229.0604
```

```
...
```

```
Iteration 6:   log likelihood =  -166.15829
```

```
Multinomial logistic regression
```

```
Number of obs   =      345
LR chi2(14)     =      125.80
Prob > chi2     =      0.0000
Pseudo R2      =      0.2746
```

```
Log likelihood = -166.15829
```

	y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
<b>2</b>						
	x1	-10.99413	1.565082	-7.02	0.000	-14.06163 -7.926625
	x2	1.959274	1.713848	1.14	0.253	-1.399806 5.318353
	x3	.00002	.0000276	0.73	0.468	-.000034 .000074
	x4	-.444847	.2259241	-1.97	0.049	-.8876502 -.0020438
	x5	20.26386	9.895417	2.05	0.041	.869197 39.65852
	x6	3.01e-06	.0003993	0.01	0.994	-.0007795 .0007855
	x7	.1615299	.1213036	1.33	0.183	-.0762209 .3992806
	_cons	.8172448	.3323678	2.46	0.014	.1658159 1.468674
<b>3</b>						
	x1	-5.229735	2.489153	-2.10	0.036	-10.10839 -.3510836
	x2	13.82778	4.370697	3.16	0.002	5.261368 22.39418
	x3	.0000185	.0000537	0.34	0.731	-.0000867 .0001237
	x4	-1.504211	.7736294	-1.94	0.052	-3.020497 .0120745
	x5	13.04574	37.47837	0.35	0.728	-60.41051 86.50199
	x6	-.000379	.000864	-0.44	0.661	-.0020723 .0013144
	x7	.3027194	.2599409	1.16	0.244	-.2067554 .8121942
	_cons	-2.047637	.8040618	-2.55	0.011	-3.623569 -.4717048

```
(y==1 is the base outcome)
```

```
. fitstat
```

```
Measures of Fit for mlogit of y
```

Log-Lik Intercept Only:	-229.060	Log-Lik Full Model:	-166.158
D(329):	332.317	LR(14):	125.804
		Prob > LR:	0.000
McFadden's R2:	0.275	McFadden's Adj R2:	0.205
ML (Cox-Snell) R2:	0.306	Cragg-Uhler(Nagelkerke) R2:	0.416
Count R2:	0.835	Adj Count R2:	0.352
AIC:	1.056	AIC*n:	364.317
BIC:	-1590.210	BIC':	-43.995
BIC used by Stata:	425.813	AIC used by Stata:	364.317

```
. mfx compute, predict(outcome(1))
```

```
Marginal effects after mlogit
```

```
y = Pr(y==1) (predict, outcome(1))  
= .87484547
```

variable	dy/dx	Std. Err.	z	P> z	[ 95% C.I. ]	X
x1	1.140146	.13885	8.21	0.000	.868009 1.41228	.27949
x2	-.3454925	.18583	-1.86	0.063	-.709721 .018736	-.046302
x3	-2.17e-06	.00000	-0.72	0.474	-8.1e-06 3.8e-06	5637.03
x4	.0603969	.02498	2.42	0.016	.011439 .109355	.15155
x5	-2.139055	1.10189	-1.94	0.052	-4.29872 .020609	.01289
x6	3.89e-06	.00004	0.09	0.928	-.000081 .000089	295.209
x7	-.0192441	.01317	-1.46	0.144	-.045059 .006571	-.073405

```
. mfx compute, predict(outcome(2))
```

```
Marginal effects after mlogit
y = Pr(y==2) (predict, outcome(2))
= .11254078
```

variable	dy/dx	Std. Err.	z	P> z	[ 95% C.I. ]	X
x1	-1.090619	.13546	-8.05	0.000	-1.35611 - .825124	.27949
x2	.1760538	.17103	1.03	0.303	-.159166 .511274	-.046302
x3	1.97e-06	.00000	0.71	0.478	-3.5e-06 7.4e-06	5637.03
x4	-.0422939	.02289	-1.85	0.065	-.087148 .00256	.15155
x5	2.005341	.98426	2.04	0.042	.076221 3.93446	.01289
x6	8.38e-07	.00004	0.02	0.983	-.000077 .000078	295.209
x7	.0157031	.0123	1.28	0.202	-.008405 .039811	-.073405

```
. mfx compute, predict(outcome(3))
```

```
Marginal effects after mlogit
y = Pr(y==3) (predict, outcome(3))
= .01261375
```

variable	dy/dx	Std. Err.	z	P> z	[ 95% C.I. ]	X
x1	-.0495276	.03451	-1.44	0.151	-.117158 .018103	.27949
x2	.1694387	.07835	2.16	0.031	.015876 .323001	-.046302
x3	2.02e-07	.00000	0.30	0.762	-1.1e-06 1.5e-06	5637.03
x4	-.0181029	.01053	-1.72	0.086	-.038739 .002534	.15155
x5	.1337143	.489	0.27	0.785	-.824706 1.09213	.01289
x6	-4.72e-06	.00001	-0.44	0.661	-.000026 .000016	295.209
x7	.003541	.00352	1.00	0.315	-.003365 .010447	-.073405

## Ordered Probit

```
. oprobit y x1 x2 x3 x4 x5 x6 x7
```

```
Iteration 0: log likelihood = -229.0604
```

```
Iteration 4: log likelihood = -185.5201
```

```
Ordered probit regression
```

```
Number of obs = 345
LR chi2(7) = 87.08
Prob > chi2 = 0.0000
Pseudo R2 = 0.1901
```

```
Log likelihood = -185.5201
```

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	-4.03138	.5938243	-6.79	0.000	-5.195255 -2.867506
x2	1.687187	.8529887	1.98	0.048	.0153596 3.359014
x3	5.56e-06	.0000117	0.47	0.636	-.0000175 .0000286
x4	-.3061237	.1149779	-2.66	0.008	-.5314762 -.0807712
x5	8.843271	4.609554	1.92	0.055	-.1912879 17.87783
x6	-.0000151	.0001689	-0.09	0.929	-.0003462 .000316
x7	.0720901	.0593954	1.21	0.225	-.0443228 .1885029
/cut1	-.2196101	.1640562			-.5411544 .1019342
/cut2	1.23118	.1943152			.850329 1.612031

Note: 1 observation completely determined. Standard errors questionable.

```
. fitstat
```

```
Measures of Fit for oprobit of y
```

Log-Lik Intercept Only:	-229.060	Log-Lik Full Model:	-185.520
D(336):	371.040	LR(7):	87.081
		Prob > LR:	0.000
McFadden's R2:	0.190	McFadden's Adj R2:	0.151
ML (Cox-Snell) R2:	0.223	Cragg-Uhler(Nagelkerke) R2:	0.304
McKelvey & Zavoina's R2:	0.471		
Variance of y*:	1.889	Variance of error:	1.000
Count R2:	0.814	Adj Count R2:	0.273

```

AIC:                1. 128      AIC*n:                389. 040
BIC:                -1592. 391   BIC':                -46. 176
BIC used by Stata:  423. 632   AIC used by Stata:   389. 040

```

```
. prchange
```

```
oprobit: Changes in Probabilities for y
```

```

x1
      Avg|Chg|          1          2          3
Mi n->Max . 3870964 . 58064461 -. 47449115 -. 10615345
      +-1/2 . 57784942 . 86677413 -. 4965061 -. 37026803
      +-sd/2 . 14522663 . 21783996 -. 19435603 -. 0234839
MargEfct . 71768335  1. 076525 -. 97409793 -. 10242709

```

```

x2
      Avg|Chg|          1          2          3
Mi n->Max . 22369273 -. 3355391 . 3049367 . 0306024
      +-1/2 . 29208588 -. 43812883 . 37244845 . 06568038
      +-sd/2 . 0465245 -. 06978673 . 06305777 . 00672901
MargEfct . 30036012 -. 45054018 . 40767307 . 04286711

```

```

x3
      Avg|Chg|          1          2          3
Mi n->Max . 2078363 -. 31175447 . 25083494 . 06091949
      +-1/2 9. 909e-07 -1. 490e-06 1. 341e-06 1. 416e-07
      +-sd/2 . 01430377 -. 02145565 . 01941167 . 00204401
MargEfct 9. 904e-07 -1. 486e-06 1. 344e-06 1. 413e-07

```

```

x4
      Avg|Chg|          1          2          3
Mi n->Max . 19223931 . 28835899 -. 2660868 -. 02227215
      +-1/2 . 05445518 . 08168274 -. 07376806 -. 00791472
      +-sd/2 . 05268519 . 07902777 -. 07137918 -. 0076486
MargEfct . 05449743 . 08174615 -. 07396833 -. 00777782

```

```

x5
      Avg|Chg|          1          2          3
Mi n->Max . 57225375 -. 85838063 . 29884491 . 55953571
      +-1/2 . 66652579 -. 99978869 . 01879035 . 98099834
      +-sd/2 . 03545871 -. 05318809 . 04808801 . 00510003
MargEfct 1. 5743165 -2. 3614748 2. 1367898 . 22468497

```

```

x6
      Avg|Chg|          1          2          3
Mi n->Max . 02960916 . 04441375 -. 04065703 -. 00375671
      +-1/2 2. 696e-06 4. 053e-06 -3. 651e-06 -3. 828e-07
      +-sd/2 . 00262523 . 00393784 -. 00356315 -. 00037469
MargEfct 2. 686e-06 4. 029e-06 -3. 646e-06 -3. 834e-07

```

```

x7
      Avg|Chg|          1          2          3
Mi n->Max . 20690374 -. 3103556 . 25215909 . 05819652
      +-1/2 . 01283323 -. 01924986 . 01741643 . 00183341
      +-sd/2 . 02055509 -. 03083265 . 02789135 . 00294127
MargEfct . 01283378 -. 01925066 . 01741904 . 00183162

```

```

Pr(y|x) . 814880791 . 175651942 . 009467283

```

```

      x1      x2      x3      x4      x5      x6      x7
x= . 27949 -. 046302 5637. 03 . 15155 . 01289 295. 209 -. 073405
sd(x)= . 203534 . 154983 14443. 5 . 967448 . 022531 977. 328 1. 60182

```

```
. predict one two three
(option pr assumed; predicted probabilities)
```

```
. g yhat=1 if one==max(one, two, three)
(32 missing values generated)
```

```
. replace yhat=2 if two==max(one, two, three)
(32 real changes made)
```

```
. replace yhat=3 if three==max(one, two, three)
```

---

(0 real changes made)

. tabulate y yhat

y	yhat		Total
	1	2	
1	255	2	257
2	51	26	77
3	7	4	11
Total	313	32	345

## Multivariate Probit Models

### Model for Financial Restructuring Strategies

#### Multivariate Probit Models

The model can be shown as

$$\begin{aligned} y_1^* &= \beta_1'x + \varepsilon_1, & y_1 &= 1 \text{ if } y_1^* > 0, 0 \text{ otherwise} \\ y_2^* &= \beta_2'x + \varepsilon_2, & y_2 &= 1 \text{ if } y_2^* > 0, 0 \text{ otherwise} \\ &\vdots & & \\ y_5^* &= \beta_5'x + \varepsilon_5, & y_5 &= 1 \text{ if } y_5^* > 0, 0 \text{ otherwise} \end{aligned}$$

$$E[\varepsilon_1 | x_1, x_2, \dots, x_6] = E[\varepsilon_2 | x_1, x_2, \dots, x_6] = \dots = E[\varepsilon_5 | x_1, x_2, \dots, x_6] = 0$$

$$\text{Var}[\varepsilon_1 | x_1, x_2, \dots, x_6] = \text{Var}[\varepsilon_2 | x_1, x_2, \dots, x_6] = \dots = \text{Var}[\varepsilon_5 | x_1, x_2, \dots, x_6] = 1$$

$$\text{Cov}[\varepsilon_i, \varepsilon_j | x_i, x_j] = \rho_{ij} \quad \forall i \neq j, i = 1, 2, \dots, 5, j = 1, 2, \dots, 5$$

#### Data

$y_{1it}$	= 1 if firm i choose asset expansion action, 0 otherwise
$y_{2it}$	= 1 if firm i choose asset reduction action, 0 otherwise
$y_{3it}$	= 1 if firm i choose debt restructuring, 0 otherwise
$y_{4it}$	= 1 if firm i choose equity issues, 0 otherwise
$y_{5it}$	= 1 if firm i choose management restructuring, 0 otherwise
$x_{1it}$	= Firm's leverage
$x_{2it}$	= Interest coverage ratio (debt burden)
$x_{3it}$	= % managerial shareholding.
$x_{4it}$	= Proportion of outside directors to total directors
$x_{5it}$	= Country's GDP growth rate
$x_{6it}$	= Firm's size

#### 5 Separate Probit Models

. probit y1 x\*, nol og

Probit regression	Number of obs	=	1543
	LR chi2(6)	=	167.40
	Prob > chi2	=	0.0000
Log likelihood = -830.24495	Pseudo R2	=	0.0916

y1	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	-.0010371	.0003794	-2.73	0.006	-.0017806 - .0002935
x2	-6.67e-08	4.52e-07	-0.15	0.883	-9.53e-07 8.19e-07
x3	.0016139	.0020492	0.79	0.431	-.0024026 .0056303
x4	.0125037	.0027853	4.49	0.000	.0070445 .0179628
x5	.0175449	.0087508	2.00	0.045	.0003937 .0346961
x6	.168635	.0210309	8.02	0.000	.1274151 .2098548
_cons	-2.035393	.1823033	-11.16	0.000	-2.392701 -1.678086

. probit y2 x\*, nol og

Probit regression	Number of obs	=	1543
	LR chi2(6)	=	15.95
	Prob > chi2	=	0.0140

Log likelihood = -670.63385 Pseudo R2 = 0.0118

y2	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	-.000246	.0002078	-1.18	0.236	-.0006532 .0001613
x2	1.07e-06	1.19e-06	0.90	0.370	-1.27e-06 3.40e-06
x3	-.0021927	.0022554	-0.97	0.331	-.0066132 .0022278
x4	.0035001	.0030096	1.16	0.245	-.0023986 .0093987
x5	-.0086695	.0084185	-1.03	0.303	-.0251694 .0078304
x6	-.0630892	.0223178	-2.83	0.005	-.1068312 -.0193472
_cons	-.6396533	.1745235	-3.67	0.000	-.9817131 -.2975936

. probit y3 x\*, nol og

Probit regression

Number of obs = 1543  
LR chi2(6) = 158.20  
Prob > chi2 = 0.0000  
Pseudo R2 = 0.0751

Log likelihood = -973.96432

y3	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	.0009642	.0001827	5.28	0.000	.0006061 .0013222
x2	1.09e-07	3.44e-07	0.32	0.751	-5.65e-07 7.83e-07
x3	-.0058828	.0019448	-3.02	0.002	-.0096946 -.002071
x4	-.0082904	.0026507	-3.13	0.002	-.0134857 -.003095
x5	.0659683	.0079619	8.29	0.000	.0503633 .0815733
x6	-.1056225	.0190819	-5.54	0.000	-.1430224 -.0682227
_cons	.5035002	.1533072	3.28	0.001	.2030236 .8039769

. probit y4 x\*, nol og

Probit regression

Number of obs = 1543  
LR chi2(6) = 24.90  
Prob > chi2 = 0.0004  
Pseudo R2 = 0.0165

Log likelihood = -743.31718

y4	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	.0001434	.0001504	0.95	0.340	-.0001514 .0004382
x2	-6.53e-07	8.81e-07	-0.74	0.459	-2.38e-06 1.07e-06
x3	-.0036262	.0021842	-1.66	0.097	-.0079072 .0006548
x4	-.0092972	.0029578	-3.14	0.002	-.0150944 -.0034999
x5	.0217562	.0086833	2.51	0.012	.0047373 .038775
x6	.042665	.0203458	2.10	0.036	.002788 .082542
_cons	-.8837326	.1645487	-5.37	0.000	-1.206242 -.5612231

. probit y5 x\*, nol og

Probit regression

Number of obs = 1543  
LR chi2(6) = 45.68  
Prob > chi2 = 0.0000  
Pseudo R2 = 0.0216

Log likelihood = -1033.2938

y5	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	-.0002979	.0001572	-1.89	0.058	-.0006061 .0000103
x2	2.81e-07	3.44e-07	0.82	0.415	-3.94e-07 9.56e-07
x3	-.0078703	.0019057	-4.13	0.000	-.0116054 -.0041352
x4	.0026619	.0025551	1.04	0.298	-.0023461 .0076698
x5	-.0217769	.0072296	-3.01	0.003	-.0359466 -.0076071
x6	.0579123	.0184321	3.14	0.002	.0217862 .0940385
_cons	-.3985652	.1464524	-2.72	0.006	-.6856066 -.1115238

### MVProbit

. mvprobit (y1 x\*) (y2 x\*) (y3 x\*) (y4 x\*) (y5 x\*)

Iteration 0: log likelihood = -4251.4541

Iteration 4: log likelihood = -4212.171

Multivariate probit (MSL, # draws = 5)

Number of obs = 1543

Wald chi2(30) = 353.46

Log likelihood = -4212.171

Prob &gt; chi2 = 0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
-----					
y1					
x1	-.000876	.0003613	-2.43	0.015	-.0015841 -.000168
x2	-5.93e-08	4.51e-07	-0.13	0.896	-9.44e-07 8.26e-07
x3	.0016751	.0020487	0.82	0.414	-.0023402 .0056904
x4	.01247	.0027881	4.47	0.000	.0070055 .0179345
x5	.0171524	.0087687	1.96	0.050	-.0000339 .0343386
x6	.172079	.0210133	8.19	0.000	.1308937 .2132643
_cons	-2.070481	.1808783	-11.45	0.000	-2.424996 -1.715966
-----					
y2					
x1	-.0002315	.0002051	-1.13	0.259	-.0006336 .0001705
x2	1.19e-06	1.23e-06	0.97	0.334	-1.22e-06 3.60e-06
x3	-.0023022	.0022554	-1.02	0.307	-.0067226 .0021183
x4	.003459	.0030047	1.15	0.250	-.00243 .0093481
x5	-.0088519	.0084227	-1.05	0.293	-.0253601 .0076562
x6	-.0632638	.0223573	-2.83	0.005	-.1070833 -.0194443
_cons	-.6366942	.1745204	-3.65	0.000	-.9787479 -.2946406
-----					
y3					
x1	.0009409	.0001781	5.28	0.000	.0005919 .0012898
x2	1.25e-07	3.52e-07	0.36	0.722	-5.64e-07 8.14e-07
x3	-.0060684	.0019535	-3.11	0.002	-.0098972 -.0022395
x4	-.008231	.002659	-3.10	0.002	-.0134425 -.0030195
x5	.0634924	.0078682	8.07	0.000	.0480709 .0789138
x6	-.1051319	.0190914	-5.51	0.000	-.1425503 -.0677134
_cons	.5159247	.1534935	3.36	0.001	.215083 .8167665
-----					
y4					
x1	.0001815	.0001552	1.17	0.242	-.0001227 .0004857
x2	-4.09e-07	6.13e-07	-0.67	0.504	-1.61e-06 7.92e-07
x3	-.0039378	.0021848	-1.80	0.071	-.00822 .0003443
x4	-.0090409	.0029429	-3.07	0.002	-.0148089 -.0032728
x5	.0197256	.0084861	2.32	0.020	.0030932 .036358
x6	.0438501	.0201694	2.17	0.030	.0043187 .0833815
_cons	-.8907171	.164368	-5.42	0.000	-1.212872 -.5685617
-----					
y5					
x1	-.000295	.000157	-1.88	0.060	-.0006028 .0000128
x2	2.65e-07	3.40e-07	0.78	0.436	-4.02e-07 9.31e-07
x3	-.0077204	.0019072	-4.05	0.000	-.0114585 -.0039823
x4	.0026275	.0025524	1.03	0.303	-.002375 .0076301
x5	-.021335	.0072211	-2.95	0.003	-.035488 -.007182
x6	.0571631	.0184448	3.10	0.002	.0210119 .0933143
_cons	-.3960648	.1465514	-2.70	0.007	-.6833003 -.1088294
-----					
/atrho21	-.0500533	.0510402	-0.98	0.327	-.1500902 .0499835
-----					
/atrho31	-.1204213	.0426728	-2.82	0.005	-.2040586 -.0367841
-----					
/atrho41	.0004555	.0471261	0.01	0.992	-.0919099 .0928209
-----					
/atrho51	.0092549	.0417803	0.22	0.825	-.0726329 .0911427
-----					
/atrho32	.0009379	.0447241	0.02	0.983	-.0867197 .0885955
-----					
/atrho42	-.0525189	.0496414	-1.06	0.290	-.1498143 .0447766
-----					
/atrho52	.1193119	.043904	2.72	0.007	.0332615 .2053622
-----					
/atrho43	.3519057	.0466562	7.54	0.000	.2604613 .4433501
-----					
/atrho53	-.0115279	.0392189	-0.29	0.769	-.0883954 .0653396
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/atrho54	-.0404692	.0422527	-0.96	0.338	-.123283 .0423446



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rho21	-.0500116	.0509125	-0.98	0.326	-.1489732	.049942
rho31	-.1198426	.04206	-2.85	0.004	-.2012726	-.0367676
rho41	.0004555	.0471261	0.01	0.992	-.091652	.0925552
rho51	.0092546	.0417767	0.22	0.825	-.0725055	.0908911
rho32	.0009379	.0447241	0.02	0.983	-.0865029	.0883645
rho42	-.0524706	.0495048	-1.06	0.289	-.1487034	.0447467
rho52	.1187489	.0432849	2.74	0.006	.0332493	.2025232
rho43	.3380646	.0413239	8.18	0.000	.254727	.4164175
rho53	-.0115274	.0392136	-0.29	0.769	-.0881659	.0652468
rho54	-.0404471	.0421836	-0.96	0.338	-.1226622	.0423193

Likelihood ratio test of  $\rho_{21} = \rho_{31} = \rho_{41} = \rho_{51} = \rho_{32} = \rho_{42} = \rho_{52} = \rho_{43} = \rho_{53} = \rho_{54} = 0$ :  
 $\chi^2(10) = 78.5662$  Prob >  $\chi^2 = 0.0000$